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ANALYSIS OF MOVEMENT OF EULER-BERNOULLI BEAM SUBJECTED TO AN IMPACT AT THE BEAM CENTER AFTER FREE DROPPING

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Abstract. *The aim of this work is to develop finite difference algorithm in order to model the vibration of a bar subjected to an impact at the bar center, simulating freely dropping. Based on by Centered Finite Difference Method (CFDM), we has discretized the general movement equation of the Euler-Bernoulli beam with the consideration of initial and boundary conditions and obtained numerical solutions of displacement and bending moment of the bar in terms of time. We also have carried out the same modeling by using the software ANSYS LS. Comparing the results from the CFDM with the one from the ANSYS, we find no significant differences between them and confirm that the algorithm developed in this work is credible, which can be used for the analysis of the crack dynamic propagation in future.*

Keywords: *Shock, Euler-Bernoulli beam, Numerical solution, Centered Finite Difference Method.*

1 INTRODUCTION

Brazil is one of major producers of iron ore in the world. After the ore is transported from extraction site to a factory, the first process of production will be the triturating. Rods Mill is most common grinding equipment. The ore is crushed by the impact with the milling bars (Fig 1). These machines are of expensive costs in maintenance due to constant fracture of the milling bars caused by the shock when a bar drops and impact with other one or ores. To reduce such costs, mathematical and mechanical analysis for the fracture is necessary.

Figure 1. Milling unit of a Rod Mill

However whether a shock can induce the bar fracture depends many factors such as the drop height, the strength, stiffness and dynamic response of the bar material as well as the position where the shock happens etc. (Anderson, 2005). When the shock comes about, the impact leads bar vibration, consequently, the induced stress in the bar varies quickly. When the stress in somewhere reaches the material strength, a crack would be initiated. If the crack propagates, the bar fracture would happen. Petroski (1984) analyzed stability of a crack at the root of a cantilever beam undergoing large plastic deformation after subjected to an impact at the free end of the beam. Also Yang and Chen (1992) studied large deflection plastic response for the same case where the crack did not propagate.

There is some commercial software that can deal with crack dynamic propagation or impact response. Unfortunately, by our knowledge, there is no such commercial software that can deal with crack propagation induced by shock or impact. Therefore this work aims to develop an efficient algorithm based on finite difference that can exactly determinate position where a crack will be initiated and could underlie the analysis of crack propagation in a vibrating beam subjected to an impact.

A vast amount of literature exists in which the finite difference form of solution has been used for vibration and buckling analysis of beams, plates and shells. Subrahmanyan and Leissa (1983) studied natural frequencies and mode shapes of uniform cantilever beams by use of the first and second order central difference schemes. The results presented is of an error equal to -0.094% for first order Mode I, compared with exact solution to flexural frequencies of uniform cantilever beam. Yu et al analyzed dynamic behavior of elastic-plastic free-free beams subjected to impact by a projectile at the mid-span using finite difference method (Yu et al., 1996) and of a moving free-free beam striking the tip of a cantilever beam (Yu et al., 2001).

2 MATHEMATICAL FOUNDAMENTALS

2.1 Governing equation of a Euler-Bernoulli´s Beam

Let´s consider a uniform beam that strikes with a rigid support at its center after free fall. The classical theory of Euler-Bernoulli is used for a uniform prismatic beam (constant cross section) with longitudinal length as the predominant dimension. In this model shear strain in the cross section is not considered. For these beams our interest is to the wave actions called bending actions. By the symmetry, we simplify the model as shown in Fig. 2. The governmental equation for the forced lateral vibration of a uniform beam (Kisa and Gurel, 2007), is given by:

$$\frac{\partial^2 M(\varsigma,t)}{L^2\partial\varsigma^2}+\rho A\frac{\partial^2 w(\varsigma,t)}{\partial t^2}=f(\varsigma,t) \qquad \varsigma=\frac{x}{L},\ 0\le\zeta\le1 \tag{1}$$

where: M is bending moment, L is half length of the rod, t is time, ρ is material density, A is the cross-sectional area, w is the transverse displacement, $f(\varsigma,t)$ is the external body force [N/m] and x is the position in the longitudinal axis, which is normalized in dimensionless by ς .

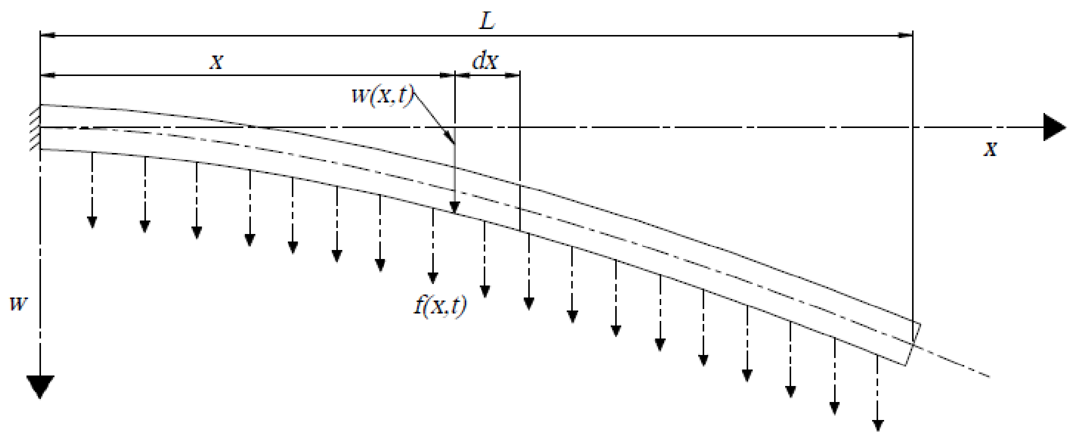

Figure 2. Lateral bending

The beam is discretized uniformly by *n + 1* points. The distance between two neighborhood point is dimensionless step $h = 1 / n$, represented as in Fig. 3.

Figure 3. Mesh

Equation (2) expresses the second derivative of M with respect to x in terms of the Central-Difference Formula:

$$\frac{\partial^2 M_i^j}{L^2\partial\varsigma^2} = EI\frac{\partial^4 w_i^j}{L^4\partial\varsigma^4} = \frac{E}{L^4h^4}(I_{i+1}^j\left[w_{i+2}^j - 2w_{i+1}^j + w_i^j\right] - 2I_i^j\left[w_{i+1}^j - 2w_i^j + w_{i-1}^j\right] + I_{i-1}^j\left[w_i^j - 2w_{i-1}^j + w_{i-2}^j\right]) \quad 1 \le i \le n \text{ and } 1 \le j \le m \tag{2}$$

where I is the moment of inertia, m number of time increments. The bending moment can be discretized by

$$M_i^j = M(\varsigma_i, t^j) = EI\frac{\partial^2 w(\varsigma_i, t^j)}{L^2\partial\varsigma^2} = \frac{EI}{L^2h^2}(w_{i+1}^j - 2w_i^j + w_{i-1}^j) \tag{3}$$

To calculate the acceleration $\partial^2 w(\varsigma L, t)/\partial t^2$, we use the formula of linear motion of a material point with constant acceleration with very short temporal increment: $\Delta t = t^j - t^{j-1}, j = 1,2,...,m$. Thus, through the relationship:

$$\Delta s = v\Delta t + \frac{1}{2}a\Delta t^2 \tag{4}$$

we get:

$$a = \frac{2(\Delta s - v\Delta t)}{\Delta t^2}. \tag{5}$$

In discretized form:

$$a_i^{j-1} = \frac{2(\Delta s_i - v_i^{j-1}\Delta t)}{\Delta t^2} \tag{6}$$

Replacing $\Delta s_i = w_i^j - w_i^{j-1}$ in Eq. (7), then we have:

$$\rho A\frac{\partial^2 w(\varsigma_i, t^j)}{\partial t^2} = \frac{2\rho A}{\Delta t^2}\left(w_i^j - w_i^{j-1} - v_i^{j-1}\Delta t\right) \tag{7}$$

Substituting Eqs. (2) and (7) into Eq. (1) and using $f(\varsigma, t) = \rho \mathrm{Ag}$, we have:

$$\frac{E}{L^4h^4}\left(I_{i+1}^j\left[w_{i+2}^j - 2w_{i+1}^j + w_i^j\right] - 2I_i^j\left[w_{i+1}^j - 2w_i^j + w_{i-1}^j\right] + I_{i-1}^j\left[w_i^j - 2w_{i-1}^j + w_{i-2}^j\right]\right) + \frac{2\rho A}{\Delta t^2}\left(w_i^j - w_i^{j-1} - v_i^{j-1}\Delta t\right) = \rho A g \tag{8}$$

After obtaining w_i^j, we can calculate a_i^{j-1} by Eq. (6), and then:

$$v_i^j = v_i^{j-1} + a_i^{j-1}\Delta t \tag{9}$$

2.2 Initial conditions

One of initial conditions is given by $w(x,0) = w(\varsigma,0) = 0$, so we have $w_i^0 = 0$. The another initial condition is the initial speed of the bar when it touches iron ore:

$$v_i^0 = \sqrt{2gH}\ . \tag{10}$$

where H is the falling height.

2.3 Boundary conditions

As shown in Fig. 2, the beam is considered as a cantilever beam. There are two boundary conditions at each end.

In the fixed end ($x = \varsigma L = 0$ or $i = 0$), the deflection is $w(0,t^j) = 0$ and the slope is given by $\partial w(0,t^j)/\partial x = (w_1^j - w_{-1}^j)/2h = 0$. So the boundary conditions for deflection and slope are respectively:

$$w_0^j = 0 \tag{11}$$

and

$$w_{-1}^j = w_1^j \tag{12}$$

At the free end ($\varsigma = 1$ or $i = n$) we have the moment:. $M(L,t^j) = EI\dfrac{\partial^2 w(1,t^j)}{L^2\partial\varsigma^2} = 0$. From Eq. (4) this results in leads:

$$w_{n+1}^j = -w_{n-1}^j + 2w_n^j \tag{13}$$

Also from the shear force: $V(L,t^j) = \dfrac{\partial}{L\partial\varsigma}\left(EI(1)\dfrac{\partial^2 w(1,t^j)}{L^2\partial\varsigma^2}\right) = 0$, we have:

$$w_{n+2}^j = w_{n-2}^j - 4w_{n-1}^j + 4w_n^j \tag{14}$$

2.4 Displacement coefficient matrix

After assembling the discretized equations, we get a linear equations system:

$$\begin{pmatrix} (7\alpha+\beta) & -4\alpha & \alpha & & & & & & & & & & \\ -4\alpha & (6\alpha+\beta) & -4\alpha & \alpha & & & & & & & & & \\ +\alpha & -4\alpha & (6\alpha+\beta) & -4\alpha & \alpha & & & & & & & & \\ 0 & \alpha & -4\alpha & (6\alpha+\beta) & -4\alpha & \alpha & & & & & & & \\ 0 & 0 & \alpha & -4\alpha & (6\alpha+\beta) & -4\alpha & \alpha & & & & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & & & & & \\ & & & & & & & \cdots & \alpha & -4\alpha & (6\alpha+\beta) & -4\alpha & \alpha \\ & & & & & & & \cdots & 0 & \alpha & -4\alpha & (5\alpha+\beta) & -2\alpha \\ & & & & & & & \cdots & 0 & 0 & 2\alpha & -4\alpha & (2\alpha+\beta) \end{pmatrix} \begin{pmatrix} w_1^j \\ w_2^j \\ w_3^j \\ w_4^j \\ w_5^j \\ \vdots \\ w_{n-2}^j \\ w_{n-1}^j \\ w_n^j \end{pmatrix} = \begin{pmatrix} \gamma_1^j \\ \gamma_2^j \\ \gamma_3^j \\ \gamma_4^j \\ \gamma_5^j \\ \vdots \\ \gamma_{n-2}^j \\ \gamma_{n-1}^j \\ \gamma_n^j \end{pmatrix} \tag{15}$$

where: $\alpha = \dfrac{EI}{L^4 h^4}$, $\beta = \dfrac{2\rho A}{\Delta t^2}$ and $\gamma_i^j = \rho A \left(g + \dfrac{2v_i^{j-1}}{\Delta t} \right)$.

3 RESULTS AND DISCUSSION

The simulation was performed by Matlab. The input parameters are listed in Table 1:

Table 1. Input parameters for the simulation

Parameter	Value
Modulus of elasticity (E)	180 GPa
Bar radius (R)	0.0508 m
Half length of the bar (L)	2.21 m
Density (ρ)	7850 Kg/m³
Drop height (H)	6.096 m
Time interval (Δt)	1 μs
Number of discretization (n)	221

To ensure the developed algorithm in correctness, we have compared the result from this work with the one from the software ANSYS (LS-DYNA module) where the elements BEAM161 were used.

Figure 4 shows the comparison of the results of transverse displacement of the bar free end i.e. $i = n$. The results obtained from this work show good coincidence with the ones of ANSYS, except for the peaks. After eliminating the possibility that the number of elements is not sufficient by means of comparing the results of 210 elements (green line in Fig. 4) with the ones of 630 elements (blue line in Fig. 4), we consider that the differences at the peaks may be attributed to that the ANSYS adds hourglass control (viscous or stiffness) such as the hourglass coefficient and bulk viscosity coefficient to avoid oscillation. From these comparisons we convince that the developed algorithm should be reliable.

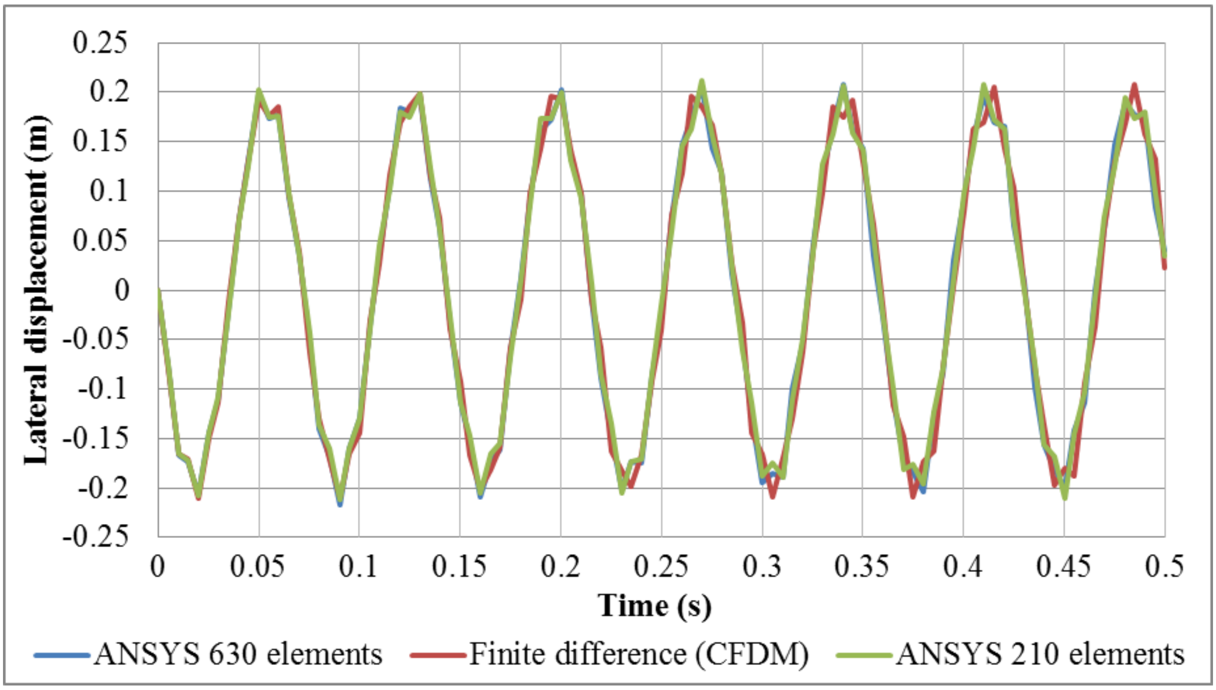

Figure 4. Displacement in the last node of the bar *i* = *n*, time from 0 to 0.5 s.

Figure 5 shows the maximum bending moment in the bar and its position. The peak between 1700 and 1800 μs can be interpreted that there are two sites about 8% and 50% of the length on the beam where the bending moments are almost identical and maximum at time 1707 μs, that is, the maximum stresses may occur in these two places at some instant. This implies that the bar may be broken down at these locations.

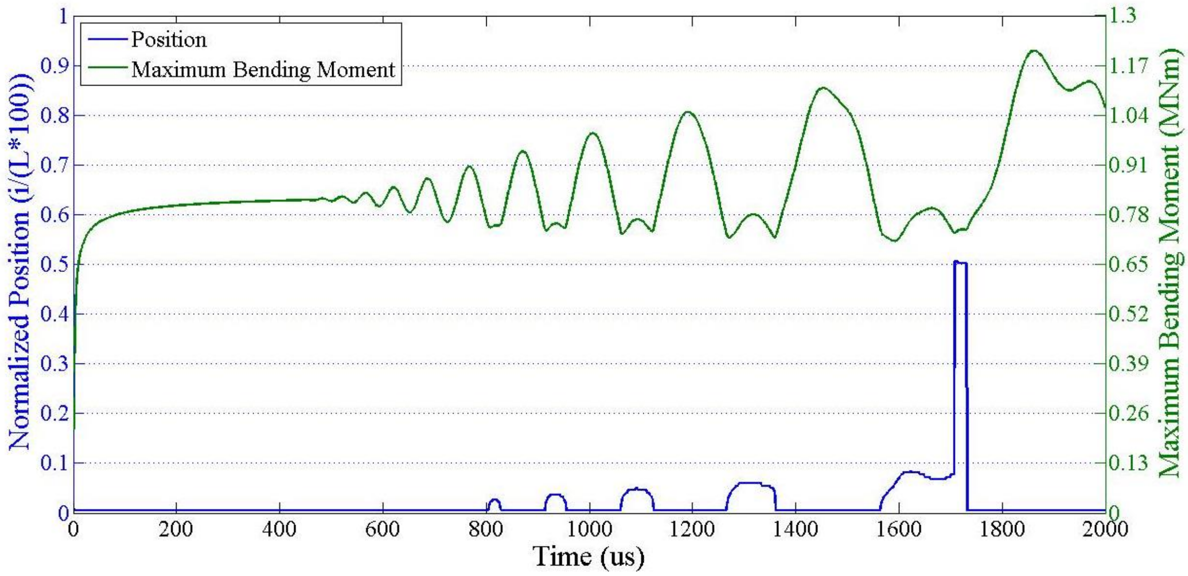

Figure 5. Maximal moment and its position vs Time.

4 CONCLUSION

Dynamic analysis of the shock effect of a general structure is complex. The computational program development based on f the finite element method (FEM) for this purpose requires a good command of the finite element method. The use of commercial software can be high cost in the acquisition as well as in the intense training. The presented algorithm based on the CFDM for analysis of Euler-Bernoulli beam is efficient and reliable, but relatively simple compared to the FEM. By the developed program we can obtain the displacement profiles, speed and acceleration of the beam at any instant and in any point of the beam. It is possible that same maximum stress may occur at more than one location in the beam at some instant. This reveals that the bar may be broken into pieces. The developed algorithm can be used in future to crack propagation analysis caused by shock.

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