NUMERICAL MODAL ANALYSIS FOR DAMAGE DETECTION IN AIRPLANE WINGS

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Abstract. *Structural Health Monitoring (SHM) has become an essential technique in Aerospace industry. The duration of service lives of such vehicles can be considerably extended by systematic monitoring of small, possibly undetected, damages to their structures. In this paper, we propose damage detection in airplane wings via modal analysis. By using Finite Elements wing models, developed in CAD software, as a first step we make comparisons of the free vibration frequencies of undamaged wing models and wing models where we deliberately introduce damage. We observe the difference in frequency values and describe numerical damage indicators based on the modal analysis.*

Keywords: *Aerospace Structure, Damage Detection, Modal Analysis*

1. INTRODUCTION

Structural Health Monitoring (SHM) has become a must in the Aerospace Industry. It serves as an indicator to when maintenance and repairs should be performed. By systematic monitoring small damages, service life of parts from a vehicle can be extended and accidents avoided. In this work, we propose the monitoring of wing damage using modal analysis, that is, determination of free vibration frequencies and modes of finite elements wing models developed in CAD software.

Computational methods allow us to easily determining free vibration frequencies and modes of wing models. The advantages are the low cost, fast execution and the easiness to change model parameters. In the other hand, Experimental Modal Analysis will also obtain the natural modes and frequencies of vibration of a structure based on field or physical models measurements, (Rodrigues, 2004).

We conduct study in an undamaged wing model and them we introduce damage along the structure. Later we compare values of the free vibration frequencies obtained and will try a damage numerical indicator in future works.

2. THEORETICAL DEVELOPMENT

2.1 Systems with Several Degrees of Freedom

In this work, we are interested in the free undamped vibrations of the wing, for in this case we are able to determinate the natural frequencies and vibration mode shapes. The equations of motion of free undamped vibrations are:

$$[M]\{\ddot{U}\} + [K]\{U\} = \{0\} \tag{1}$$

where,

- $[M]$ - System Mass Matrix.
- $\{\ddot{U}\}$ - Accelerations Vector.
- $[K]$ - System Stiffness Matrix.
- $[U]$ - Displacements Vector.

Differently from a system with only one degree of freedom, Eq. (1) is represented with matrices and vectors for we are dealing with several degrees of freedom. The order of the matrices will depend on the number of degrees of freedom adopted in the system. For instance, for a system with N degrees of freedom, the matrices will have the order of NxN and the vectors Nx1 (Juliani, 2014).

Considering the free vibration motion, in the same way as in a system with one degree of freedom, the dynamic response can be expressed as:

$$\{U(t)\} = \{\tilde{U}\}\cos(\omega t - \Theta) \tag{2}$$

Deriving for the second time Eq. (2) and substituting in Eq. (1) we generate the eigenvalues and eigenvectors problem:

$$[[K] - \omega^2[M]\{\tilde{U}\} = \{0\} \qquad (3)$$

where:

- ω^2 - Eigenvalues that represent the squares of the circular frequencies;
- $\{\tilde{U}\}$ - Eigenvectors that represent the corresponding dimensionless displacements (shape) of their respective modes (Clough and Penzien, 2003).

Equation (3) is a set of algebraic linear homogenous equations and the non-trivia solution of the equation is only possible if (Anderson and Naeim, 2012):

$$|[K] - \omega^2[M]| = 0 \qquad (4)$$

The determinant in Eq. (4) results in a N-th order polynomial equation, having as unknown the parameter ω^2. The N roots represent the natural frequencies of the N modes of vibration adopted in the system (Clough and Penzien, 2003). The smallest frequency value calculated corresponds to the first vibration mode, the second to the second vibration mode and so on, successively.

To obtain the vibration modes we solve:

$$[E^n]\{\tilde{U}_n\} = \{0\} \qquad (5)$$

where

$$[E^n] = [K] - \omega_n^2[M] \qquad (6)$$

Matrix $[E^n]$ varies with each frequency and so it is different for each mode. This way, for each ω_n, a vibration mode may be calculated with Eq. (5), but, in this system, one of the equations is redundant, so the vibration modes are found by arbitrarily giving an unitary value to one of the displacements.

With the advances of computation and the use of programs, all the formulation presented here can be rapidly solved using iterative methods.

3. DAMAGE DETECTION BY MODAL ANALYSIS

In this work, we make use of a CAD wing model, for which we determine the free vibration modes with the help of a computer software. Then we use the software to determine the free vibration modes on the same wing model but this time with damage introduced to its structure. To identify and locate the damage, later we will use four techniques which are the MAC (Modal Assurance Criterion), the COMAC (Coordinate Modal Assurance Criterion), the MCD (Modal Curve difference) and the DI (Damage Index), presented in this section.

3.1 Free Vibration Determination

Determination of free vibrations performed by the software is similar to an experimental liberating the structure to vibrate after applying a load and immediately removing it. This way we can obtain the free vibration modes and natural frequencies.

3.2 Analysis Criterion

The MAC coefficient offers a consistence measure between estimated modal vectors (Allemang, 2003). This estimative can be obtained by the Finite Elements Method, for instance. We use as referential the non-damaged situation to compare it with the damaged one. We can also compare modal vectors obtained via mathematical model and in this case the reference is the theoretical non-damaged model.

The MAC coefficient may be obtained with the expression:

$$MAC_{(i,j)} = \frac{|\{\phi_i^a\}^T\{\phi_j^e\}|^2}{(\{\phi_i^a\}^T\{\phi_i^a\})(\{\phi_j^e\}^T\{\phi_j^e\})} \tag{7}$$

where,

- $\{\phi_i^a\}$: i mode analytical modal vector;
- $\{\phi_j^e\}$: j mode modal experimental vector.

The MAC coefficient relates pairs of modal vectors assuming values between 0 and 1. The value 1 indicates that the vectors are the same and have great correlation; while the value 0 indicates that the compared modal vectors are orthogonal, having no correlation whatsoever (Farrar and Worden, 2013).

The COMAC coefficient correlates two modal vectors for each degrees of freedom, being one of them the reference condition. It is an extension of the MAC coefficient that tries to identify the degrees of freedom which negatively contributes to the MAC value (Allemang, 2003).

In each j degrees of freedom, the COMAC is calculated for each mode following the expression:

$$COMAC_{(j)} = \frac{(\sum_{i=1}^{n}\{\phi_i^a\}_j\{\phi_j^e\}_j)^2}{(\sum_{i=1}^{n}\{\phi_i^a\}_j\{\phi_i^a\}_j)\sum_{i=1}^{n}\{\phi_j^e\}_j\{\phi_j^e\}_j)} \tag{8}$$

where,

- $\{\phi_i^a\}$: reference i mode modal vector;
- $\{\phi_j^e\}$: j mode experimental modal vector;
- n: number of vibration modes.

The COMAC also gives results between 0 and 1, and their meaning is the same as in the MAC coefficient but the modal vectors are related to a degree of freedom. So the COMAC coefficient can be understood as an investigation in a determined degree of freedom. Low correlation of a degree of freedom may indicate a localized damage.

The Modal Curve Difference method is based in the fact that a damage on the structure alters its dynamics properties. For instance, the natural frequencies and modal shapes. Besides, a fissure or a located damage also lower the stiffness (and also raises the damping of the structure). This occurs in lower natural frequencies and modify the vibration modes of the structure (Juliani, 2014).

For structures that bend, a parameter to identify damage is the MCD. The difference of curvature between the non-damaged and the damaged locate a possible damaged region in the structure (Pandey, *et al*, 1991).

To obtain the curvatures, we numerically perform the second derivate of the modal shape. The advantage of the curvature is that the derivative process has the effect of amplifying an eventual discontinuity in the modal deformity caused by a located damage (Farrar and Worden, 2013).

The curvatures of the modal shapes are related to the flexional stiffness of the transversal sections of the beams. Analytically, they can be calculated in a *x* section as follows (Pandey, *et al*, 1991):

$$\Phi''(x) = \frac{M(x)}{EI} \tag{9}$$

where,

- M(x): flexion moment of the x section;
- E: Elasticity mode of the beam;
- I: Moment of inertia of the beam transversal section.

If a fissure or another damage is present in the structure, the EI term of Eq. (9) lower´s and the curvature in the x section is raised. The larger the change in the curvature, the greater the damage on the structure.

Performing the difference in the curvatures between the non-damaged and the damaged situation, in the *x* section there will be a peak indicating the damage. In the graph, the axis abscises is the length of the beam and the ordinate axis the absolute value of curvatures difference.

The Damage Index (DI) is based on the curvatures of the modal shapes and in the energy of the modal deformation, which is a strain energy stored in the structure when it deforms in a vibration mode (Stubbs, *et al*, 1992).

When damage occurs, the strain energy distribution originally stored in a structure will be considerably altered in the damaged regions. As a structural element has a reduction in its stiffness caused by the damage, this element won´t be able to store the same amount of energy as the non-damaged structure (Farrar and Worden, 2013).

Based on these formulations and in the fact that the curvature is the second derivate of the modal shapes, Stubbs, Kim and Farrar (1995) present the DI equation:

$$B_{ij} = \frac{(\int_j^{j+1}[\phi_j^{d\prime\prime}(x)]^2 dx + \int_0^l[\phi_j^{d\prime\prime}(x)]^2 dx)\int_0^l[\phi_j^{\prime\prime}(x)]^2 dx}{(\int_j^{j+1}[\phi_j^{\prime\prime}(x)]^2 dx + \int_0^l[\phi_j^{\prime\prime}(x)]^2 dx)\int_0^l \phi_j^{d\prime\prime}(x)]^2 dx} \tag{10}$$

where,

- $\phi_j^{\prime\prime}(x)$: is the second derivate of the j vibration mode in the non-damaged situation;
- $\phi_j^{d\prime\prime}(x)$: is the second derivate of j vibration mode in the damaged situation.

The results are often presented as a graph where the abscises axis is the length of the beam or the measure spots positions and the ordinate axis are the DI values (B_{ij}).

4. WING MODELS

Figure 1 shows the non-damaged wing model being used in this work. It was made using a CAD software that allows the attribution of the material, with its respective properties.

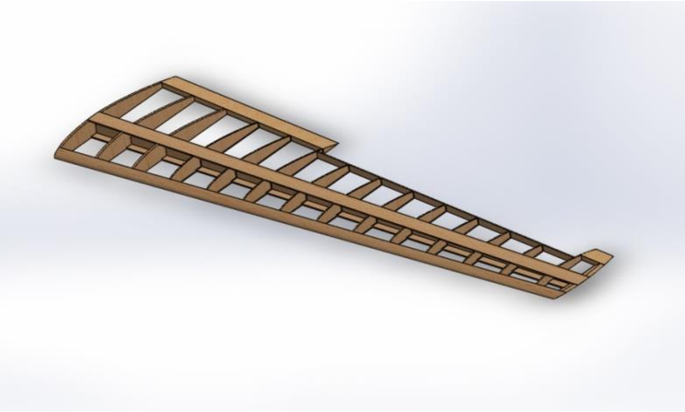

Figure 1.CAD non-damaged wing model.

Figure 2 shows the damaged wing model being used in this work. We can see the damage introduced in the upper part of the center beam near the binding side

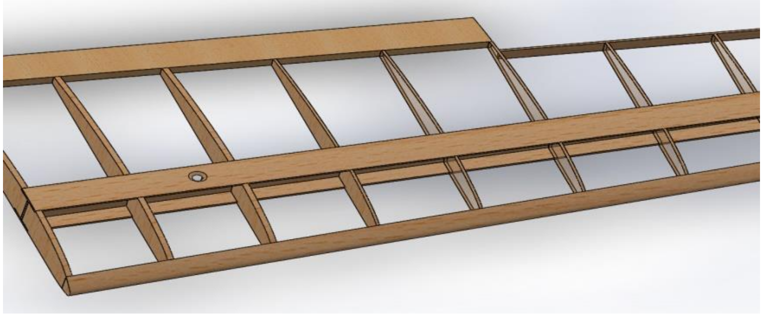

Figure 2. CAD damaged wing model with hole near the binding spot.

Figure 3 shows the damaged wing model now with a hole about the center of the beam

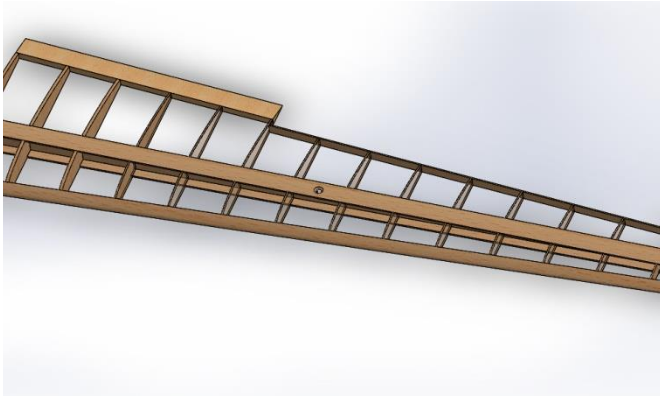

Figure 3. CAD damaged wing model with hole on the center region of the beam.

Figure 4 shows the damaged wing model with a hole on the side of the wing which is free (absent of binding).

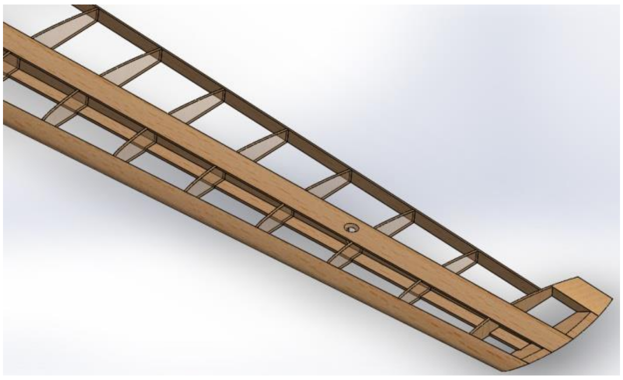

Figure 4. CAD damaged wing model with hole on the free side of the wing.

All wings are modeled in balsa wood, which is the same material used to build wings for airplane models.

5. RESULTS

With the help of the software, we calculated the free vibration modes and their natural frequencies. For this we considered the material of the wings to be balsa wood and that the larger side of the structure was bonded to the body of the airplane. The elasticity modulus of the balsa wood used in all the situations is 3000 N/mm^2. The results of the natural frequencies are presented in Tab. 1 for the first five free vibration modes.

Table. 1 Natural Frequencies for the free vibration modes of the non-damaged and the damaged wing models.

Modes	Natural Frequencies (Hz)			
	Non-Damaged Wing	Damage Near the Binding Spot	Damage on the Center	Damage on the Free Side
1	44.878	47.035	49.423	53.517
2	92.637	105.38	96.383	110.68
3	134.82	153.69	160	173.55
4	179.47	180.99	187.86	200.22
5	233.76	254.93	240.1	271.12

Figure 5 shows the number of nodes, elements and degrees of freedom considered in the models when solving for the modes

No. of nodes	15076
No. of elements	6890
No. of DOF	113136

Figure 5. Nodes, elements and degrees of freedom (DOF) of the wing models.

6. CONCLUSIONS

As expected, the natural frequencies of the damaged wings present different values them those of the non-damaged wing. Also, as the damage moves toward the tip of the wing on the free side, the amplitude of the modes rises, except on the case of the center damage where the second and fifth modes are different from this pattern. As discussed before, this happens because the dynamic properties of the structure are altered when it is damaged, and the stiffness is changed. For further analysis, we pretend to use the analysis criterion described in this paper to see the accuracy of such information and later develop a numerical analysis for damage detection. We also intend to make analysis on reduced-scale models using electronic data gathering devices.

7. ACKNOWLEDGEMENTS

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