Total Harmonic Distortion Indices Assessment Using the Maximal Overlap Discrete Wavelet Packet Transform

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***Abstract*—This paper proposes the estimation of the root mean square (RMS) values (voltages and currents) as well as total harmonic distortion (THD) using the Maximal Overlap Discrete Wavelet Packet Transform (MODWPT). The performance of the proposed method was assessed and compared with the IEEE Standard 1459-2010 by performing two case studies under stationary power quality disturbance originated by linear and non-linear loads. As an advantage, the MODWPT provides uniform frequency bandwidths, as well as possess time-invariant property, which is ideal for analyzing instantaneous power quality indices in real-time.**

***Index Terms*—Wavelet packet, power quality, total distortion harmonic.**

I. Introduction

Some of the most known important disturbances that affect the power quality (PQ) in utilities and end users are voltage and current harmonic distortions. Harmonic distortion is caused by nonlinear devices in the power system [1]. A nonlinear device can be interpreted when the current is not proportional to the applied voltage. Therefore, these nonlinear devices inject harmonic currents in the power system, deteriorating the sinusoidal wave shape.

Traditional methods based on the RMS value of voltages and currents as well as the THD have been performed both in time and frequency domain using the Fourier transform (FT) and its modifications and improvements [2]. The Fourier transform provides only magnitude frequency, whereas the time or space information is lost. In addition, the FT spectrum can lead to incorrect results caused by some problems, such as aliasing effect, spectral leakage, and picket-fence effect, as well as a higher computational burden [3].

To overcome the limitations of the FT, the discrete Fourier transform (DFT) has been widely used for analyzing and measuring both stationary and nonstationary signals in power system harmonics. Traditionally, the DFT method has been widely applied due to its simplicity, and it can be efficiently calculated using fast Fourier transform (FFT) [4]. However, in non-sinusoidal conditions, the application of the DFT method for spectral analysis can lead to wrong interpretations due to the spectral leakage and picket-fence effects [5].

According to the IEEE Standard 1459-2010, several PQ indices, such as RMS value, power (reactive, active, apparent), power factor, and harmonic distortion have been proposed to analyze systems and circuits under sinusoidal and nonsinusoidal conditions based on FT. These indices are the main fashion to quantify and monitor the severity of the harmonic distortions present in power systems. However, the FT can provide wrong results in the presence of both stationary and nonstationary PQ disturbances.

The wavelet transform (WT) is a powerful mathematical tool used for analyzing both stationary and nonstationary signals. In this context, the WT has been widely utilized to estimating PQ indices. In [6], [7], the method based on discrete wavelet transform (DWT) is proposed for estimating RMS values and power components (active, reactive and apparent power). Regarding measuring harmonic indices, several works have also been proposed in the literature. In [8], [9] a combination of two versions of the wavelet analysis (DWT and continuous WT) for estimating and quantifying harmonic contents was proposed. In addition, the authors have also developed a method to compensate the distortion introduced by low- and high-pass filters. In [10]–[12], an approach based on DWT was applied for THD calculation. However, the DWT cannot be used for harmonic analysis, since the frequency output bands do not have the same bandwidth [2], [13].

To overcome the problems of the DWT, the discrete wavelet packet transform (DWPT) has been proposed. The DWPT is a generalization of the conventional DWT algorithm, where both scaling and wavelet coefficients are decomposed [13], [14]. In [2], [15], the DWPT was applied for analyzing RMS values and amplitude of harmonics. As an advantage, DWPT provides a uniform frequency band, which gives better results than DWT for measuring harmonics [16]. However, both DWT and DWPT are time-variant transformation, which may lead to problems in online measurement for PQ analysis.

In this context, the maximal overlap DWPT (MODWPT) can be employed for solving the main problem present of DWT and DWPT, since the MODWPT provides uniform frequency bandwidths as well as possess time-invariant property.

In this paper, a real-time version of the MODWPT is used for estimating instantaneous RMS values and voltages and currents THD at an electrical system. The proposed MODWPT-based method for measuring RMS values and THD was assessed and compared to the IEEE Standard 1459-2010 using the DFT, in the real-time. The experimental results were obtained by means of actual case studies, including linear and non-linear loads. As an advantage, the proposed method provides monitoring and an accurate prediction of the harmonic distortion in power systems. Therefore, the proposed method is ideal for monitoring and quantifying the impact of a poor PQ.

II. Wavelet Transform

The DWT can be implemented through the Mallat algorithm with low- and high-pass filters and down-sampling by a factor of two to decompose a signal into different subbands or levels. Fig. 1(a) depicts the process of decomposition for a sampling frequency (f_s) of the original signal x using a three-level decomposition tree for the DWT. The blocks g and h represent the low- and high-pass filters, whereas the blocks with $\downarrow 2$ represent the operation of down-sampling by a factor of 2. The wavelet and scaling coefficients (s and ω) in Fig. 1(a) are the outputs of the low- and high-pass filters, and represent the high- and low-frequency components, respectively. Fig. 1(b) depicts the ideal frequency response to a sampling frequency of 1920 Hz of the input signal.

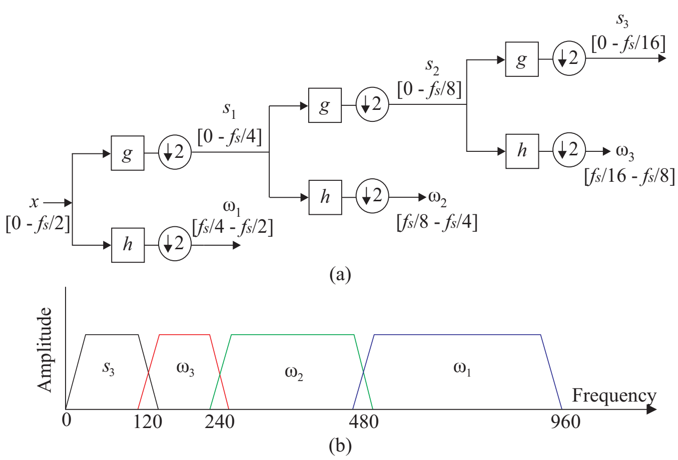

Fig. 1. DWT decomposition: (a) three-level decomposition tree; (b) ideal frequency response.

According to Fig. 1(b), DWT provides non-uniform outputs bandwidth. In contrast to the DWT, DWPT promotes uniform frequency bands, which is ideal to estimate RMS values and THD components. The DWPT is an extension of the traditional DWT algorithm. In the DWT, at any level of decomposition, only the scaling coefficients (low-frequency components) are further decomposed, whereas in the DWPT both the scaling and wavelet coefficients (low- and high-frequency components) are decomposed. However, due to the down-sampling process, both DWT and DWPT are time-variant transformations, i.e, the scaling and wavelet coefficients are computed in alternate samplings due to the process of the down-sampling by a factor of 2.

In order to overcome the DWT and DWPT limitations, MODWPT can be used to obtain a uniform frequency band, whereas the property of time-invariance is preserved. Therefore, the MODWPT can be properly used for real-time analysis purposes, such as: quantifying and monitoring harmonic distortions. In real time, the coefficients of the MODWPT, at any level j, are obtained by the convolution of the original signal x with the filters $g(l)$ and $h(l)$, respectively, as follows [17]:

$$s_j^{2z}(k) = \frac{1}{\sqrt{2}} \sum_{l=0}^{L-1} g(l) s_{j-1}^{z}(k+l-L+1), \quad (1)$$

$$s_j^{2z+1}(k) = \frac{1}{\sqrt{2}} \sum_{l=0}^{L-1} h(l) s_{j-1}^{z}(k+l-L+1), \quad (2)$$

where L represents the length of the filter and z is the node number in the wavelet packet decomposition tree.

III. Harmonic Indices

In this section are presented the definitions of RMS values and voltage and current THD contained in the IEEE Standard 1459-2010 [18].

A. The RMS Value

Based on [18], the instantaneous voltage $v(t)$ and current $i(t)$ versus time are defined as:

$$v(t) = V_0 + \sqrt{2} \sum_{h \neq 0}^{\infty} V_h \sin(h\omega t + \alpha_h), \quad (3)$$

$$i(t) = I_0 + \sqrt{2} \sum_{h \neq 0}^{\infty} I_h \sin(h\omega t + \beta_h), \quad (4)$$

where V_0 and I_0 are the average values of the voltage and current, V_h and I_h represent the RMS values of voltage and current harmonic of h order; α_h and β_h are the phase angles of the voltage and current harmonic of h order.

The effective of RMS values of non-sinusoidal voltage and current are defined as:

$$V = \sqrt{\sum_{h=0}^{\infty} V_h^2}, \quad (5)$$

$$I = \sqrt{\sum_{h=0}^{\infty} I_h^2}. \quad (6)$$

According to [18], voltage and current RMS values are defined in terms of the fundamental component and harmonic components as:

$$V^2 = V_1^2 + V_H^2, \quad (7)$$

$$I^2 = I_1^2 + I_H^2, \quad (8)$$

where

$$V_H^2 = \sum_{\forall h \neq 1} V_h^2, \quad (9)$$

$$I_H^2 = \sum_{\forall h \neq 1} I_h^2, \quad (10)$$

where V_1 and I_1 represent RMS values of the voltage and current of the fundamental component.

B. Total Harmonic Distortion

According to the IEEE Standard 1459-2010, the overall deviation of a distorted wave of its fundamental can be estimated from the THD. The THD is the most common index used for measuring the harmonic content of a signal [19]. Therefore, THD is the measure of the wave distortion degree in relation to the waveform sinusoidal.

The THD can be defined in terms of two distinct components: 1) the current THD (THD_I), which is defined as the ratio of the total harmonic RMS current and the fundamental RMS current; 2) the voltage THD (THD_V), which is defined as the ratio of the total harmonic RMS voltage and the fundamental RMS voltage. The THD_V can be computed as follows:

$$THD_V = \frac{\sqrt{\sum_{h \neq 1} V_h^2}}{V_1}, \quad (11)$$

similarly, the THD_I is computed as follows:

$$THD_I = \frac{\sqrt{\sum_{h \neq 1} I_h^2}}{I_1}. \quad (12)$$

IV. The proposed MODWPT-based RMS values and Total Harmonic Distortion

This section presents the proposed MODWPT-based method for estimating the RMS values and THD of the voltages and currents, which is in accordance with the IEEE Standard 1459-2010 [18].

A. The RMS value

The RMS values of the voltage and current using the MODWPT are defined as following [17]:

$$V(k) = \sqrt{(V_j^0(k))^2 + \sum_{z=1}^{2^j-1} (V_j^z(k))^2}, \quad (13)$$

$$I(k) = \sqrt{(I_j^0(k))^2 + \sum_{z=1}^{2^j-1} (I_j^z(k))^2}, \quad (14)$$

where

$$V_j^0(k) = \sqrt{\frac{1}{2N} \sum_{n=k-N+1}^{k} \sum_{l=0}^{L-1} \left[s_{j,v}^0 (k-l)\right]^2}, \quad (15)$$

$$V_j^z(k) = \sqrt{\frac{1}{2N} \sum_{n=k-N+1}^{k} \sum_{l=0}^{L-1} \left[s_{j,v}^z (k-l)\right]^2}, \quad (16)$$

$$I_j^0(k) = \sqrt{\frac{1}{2N} \sum_{n=k-N+1}^{k} \sum_{l=0}^{L-1} \left[s_{j,i}^0 (k-l)\right]^2}, \quad (17)$$

$$I_j^z(k) = \sqrt{\frac{1}{2N} \sum_{n=k-N+1}^{k} \sum_{l=0}^{L-1} \left[s_{j,i}^z (k-l)\right]^2}, \quad (18)$$

where N is the period, $V_j^0(k)$ and $I_j^0(k)$ represent RMS values of the voltage and current of the lowest frequency band, i.e., at node zero and level j, whereas $V_j^z(k)$ and $I_j^z(k)$ represent RMS values of the voltage and current of the higher frequency bands, i.e., at node $z \neq 0$.

B. Total Harmonic Distortion

The THD_V and THD_I using the MODWPT-based method are defined as follows [17]:

$$THD_V(k) = \frac{\sqrt{\sum_{z=1}^{2^j-1} (V_j^z(k))^2}}{V_j^0(k)}, \quad (19)$$

$$THD_I(k) = \frac{\sqrt{\sum_{z=1}^{2^j-1} (I_j^z(k))^2}}{I_j^0(k)}. \quad (20)$$

V. Performance Evaluation

This section considers two case studies in order to evaluate the proposed MODWPT-based estimation of RMS value and THD. The case studies include stationary power quality disturbances for linear and non-linear loads. The obtained results with IEEE Standard 1459-2010 definitions based on DFT were compared to the proposed MODWPT method implemented using the mother wavelet of the family Daubechies with 30 coefficients (db(30)), as well as a three-level decomposition tree. Based on [17], db(30) is suggested when harmonic distortions are desired. In addition, the db(30) provide satisfactory and accurate results in the estimation of RMS values of harmonic components

Fig. 2 presents the implemented single-phase system. The experimental platform is composed by a sinusoidal voltage and its internal impedance ($r_s = 0.6\Omega$ and l_s= 300 μH), a non-linear load implemented by a single-phase rectifier feeding a *RL* load ($r_{nl} = 40\Omega$ and l_{nl}= 30 mH) by the closed switch (K1) and a linear load ($r_l = 20\Omega$ and $l_l = 60$ mH) by the closed switch (K2). The proposed method and the IEEE Standard 1459-2010 were implemented in dSPACE 1103 data acquisition system to be evaluated in the real-time. The adopted sampling frequency was 1920 Hz (32 samples per 60 Hz fundamental cycle). The voltages and currents were measured through sensors based on Hall effects, data acquisition devices, and A/D converters.

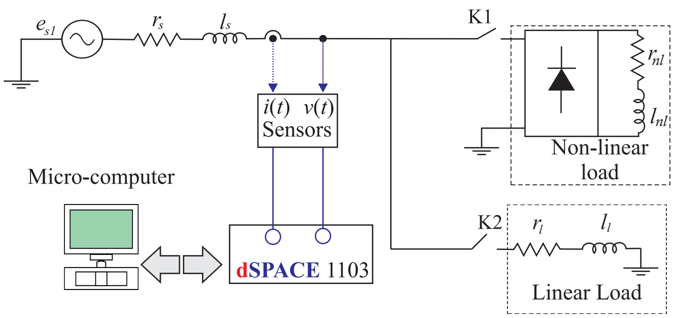

Fig. 2. Experimental power system setup.

A. Stationary PQ Disturbances: Non-Linear Load

In order to analyze the harmonic distortion caused by non-linear load behavior, the switch K1 is kept closed, whereas switch K2 is opened. Fig. 3 depicts the domain voltage and current waveforms (5 cycles). The voltage presents the low rate of harmonic content, whereas the current presents considerable harmonic content.

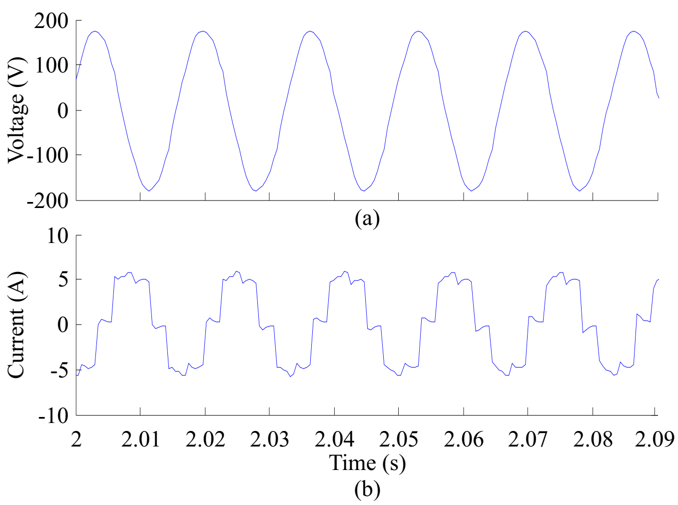

Fig. 3. Stationary PQ Disturbances: (a) voltage; (b) current.

Fig. 4 depicts the RMS voltages and currents, as well as THD_V and THD_I, in real-time, obtained by the IEEE Standard 1459-2010 and the proposed method with db(30) mother wavelet. According to Fig. 4(c-d), the rate of THD_V is less than 5%, whereas due to the rectifier effect, the rate of THD_I is higher than 30%. Besides, the estimations of RMS voltages and currents, Fig. 4(a-b), as well as THD_V and THD_I were similar in both approaches. However, the proposed MODWPT-based method presented a time delay due to the high-order mother wavelet adopted.

Table I summarizes the average values of the waveforms depicted in Fig. 4. For comparison purposes, the PQ indices estimation based on the IEEE Standard 1459-2010 is also carried out. The IEEE Standard was considered as the reference value.

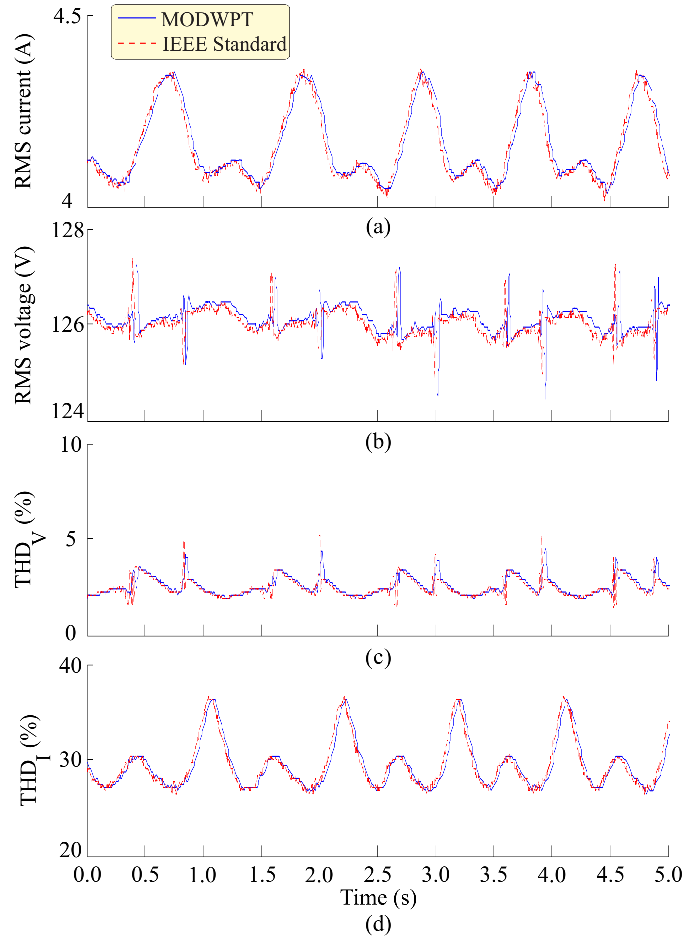

Fig. 4. Real-time estimation: (a) rms current; (b) rms voltage; (c) THD_V; (d) THD_I.

The estimated PQ indices are: current RMS, voltage RMS, THD_V and THD_I. The difference is very small between both techniques analyzed. However, a slightly larger difference was obtained in the estimation for THD_V.

TABLE I
AVERAGE VALUES OBTAINED FROM APPLYING THE IEEE STANDARD AND MODWPT.

	IEEE Std. 1459-2010	MODWPT	Difference (%)
Current RMS	4.15	4.16	0.24
Voltage RMS	126.00	126.12	0.10
THD_V	2.44	2.51	2.75
THD_I	29.56	29.54	0.10

B. Stationary PQ Disturbances: Linear and Non-Linear Loads

In the second case study, the same experimental setup of Fig. 2 was used. However, the switch K2 is kept closed providing a new load configuration. Fig. 5 depicts the time-domain voltage and current waveforms around 0.1 s for linear and non-linear loads connected in the electrical system. Fig. 6

depicts the system RMS voltages and currents while feeding linear and non-linear loads, as well as THD_V and THD_I in real-time obtained by the IEEE Standard 1459-2010 and the proposed MODWPT.

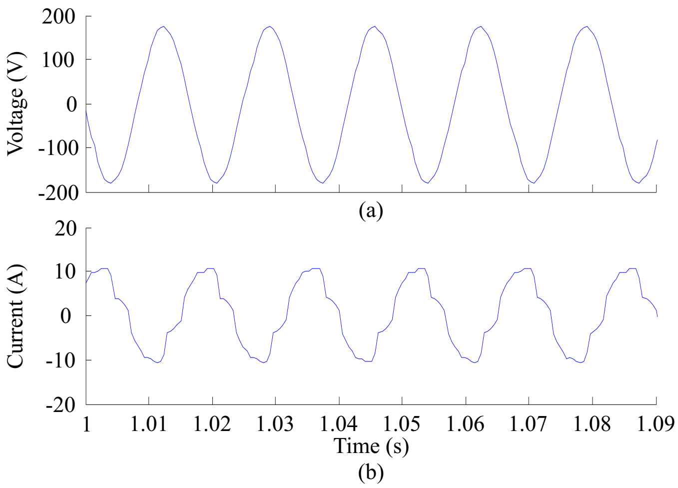

Fig. 5. Stationary PQ Disturbances: (a) voltage; (b) current.

According to Fig. 6 (a-b), the RMS currents and voltages were similar and there is no significant difference between the average current and voltage values summarized in Table II. As in the first case study, the PQ indices estimation are also compared with IEEE Standard since the true values of PQ indices are unknown. The percentage differences are also very small in the two approaches analyzed, even though that THD_V estimation difference keeps the highest difference with average values around 2.09%.

TABLE II
AVERAGE VALUES OBTAINED FROM APPLYING THE IEEE STANDARD AND MODWPT.

	IEEE Std. 1459-2010	MODWPT	Difference (%)
RMS Current	7.59	7.59	0.00
RMS Voltage	124.87	124.86	0.10
THD_V	2.25	2.55	2.09
THD_I	15.30	15.33	0.20

The performance of the MODWPT-based PQ indices measurement can be considered as an alternative tool in real-time monitoring harmonic distortions in power systems.

VI. Conclusion

The real-time estimation of RMS values and voltage and current total harmonic distortion was proposed in this paper using the maximal overlap wavelet packet transform. As an advantage, the proposed method provides uniform frequency bandwidths, as well as possess time-invariance property, which is ideal for analyzing instantaneous power quality indices in real-time.

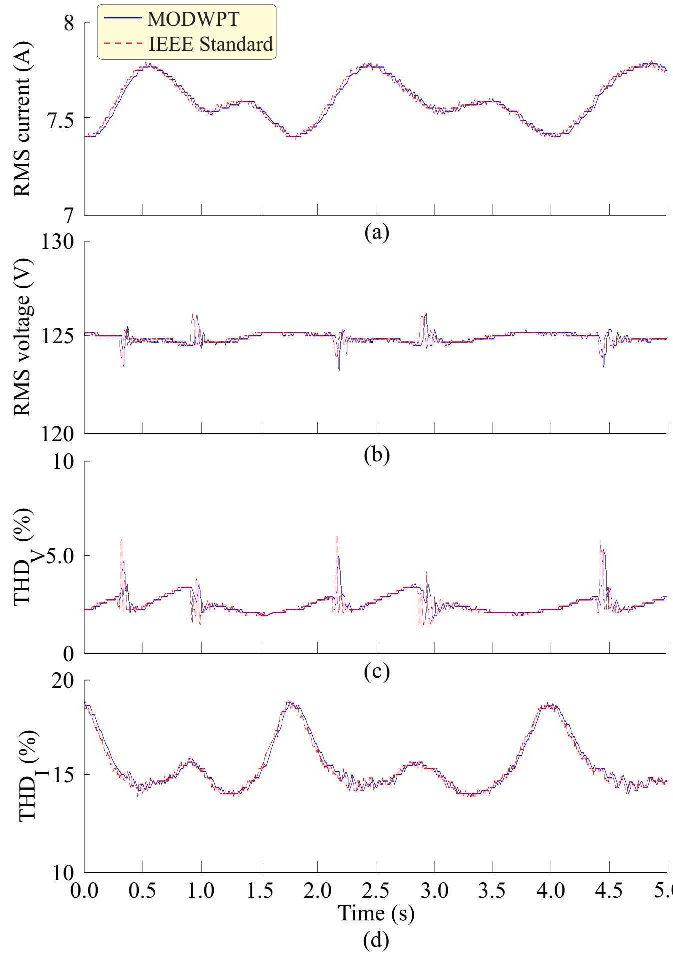

Fig. 6. Real-time estimation: (a) RMS current; (b) RMS voltage; (c) THD_V; (d) THD_I.

The results obtained by the proposed method were compared to the definitions contained in IEEE Standard 1459-2010, including two case studies in an experimental. The proposed method presented very similar performance to the DFT method in experimental results. Therefore, the proposed wavelet-based method can be an appropriate tool for monitoring and mitigating PQ indices in real-time in power systems.

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