Optimal placement of capacitor banks and voltage regulators on active distribution systems using a goal programming approach

***Abstract*— Power distribution systems (PDS) are susceptible to energy loss and voltage drops mainly because of the way that they are operated and built. One way to minimize system loss and improve the voltage profile of PDS is with the use of capacitor banks and voltage regulators (VRs) along the distribution feeders. In this scenario, the optimal placement of these devices becomes crucial. In this paper, a quadratic multi objective goal programming method is proposed to obtain the optimal placement of these devices, where the goals are energy loss and the total investment cost. The increase of distributed generation penetration brings new challenges for control and operation of PDS, therefore the effects of these power sources are considered.**

***Keywords*—Distributed generation, distribution systems, energy losses, goal programming, optimal placement.**

I. Introduction

Due to the policy of reducing costs and the constant increase in demand from regulators and consumers for a better quality of service, the energy utility has continually invested in new techniques and devices in order to improve their service. One of the parameters that measures the quality of service delivered to consumers is the voltage level. The energy loss is another important variable for distribution power companies because it directly affects their annual revenue. The majority of the active power loss occurs in PDS, representing about 15% of the total generated energy, which means a waste of millions of dollars. One way to reduce system loss and also improve voltage pattern is the use of equipment such as capacitor banks and VRs along the distribution feeders [1]. The capacitor banks (fixed or switched) can provide locally, part of the reactive power demand by loads, decreasing the reactive part of the current. Still, VRs can change the voltage level on the system buses by changing the tap level [2]. The problem of optimal allocation of capacitor banks is to determine the position, quantity, size, type and stage (for the switched capacitor banks). For VRs, the amount, position and tap control should be determined. Various methods for solving this problem have been proposed over the years, and they can by divided into four categories: deterministic, numerical, heuristic and, more recently, meta-heuristic [3]. These methods are distinguished by the use or not of power flows in the calculation of system voltages and currents, the different formulations of the objective function and constraints, consideration of different devices and different optimization methods. Some of these methods cannot guarantee the optimal solution due to the existence of non-linearity and non-convexity of the objective function or constraints [4].

Recently the use of distributed generation (DG), such as solar and wind, connected directly to the distribution system has greatly increased. In this study, the effect of this connection on technical loss and voltage profile of PDS will be evaluated. Distributed generation can change the power flow in the network and thus can change grid losses. If a small generator is allocated close to a big load, the network loss can potentially be reduced because the active and reactive power can be supplied to the load from the adjacent generator. However, if a large distributed generator is allocated away from the network loads, it will probably increase the losses in the distribution system. In general, there is a correlation between high loads on the distribution network and the use of power plants. Therefore, any generator operating in that period can make a significant impact on network operations and on its costs. In this work, only the active power contribution was considered.

A review of the methods used in capacitor placement is shown in [3] and [19]. The majority of papers in literature use backward/forward sweep method or Newton power flow methods, limiting the optimization formulation problem in commercial products. In addition, most of papers uses a meta-heuristic methodology to allocate capacitor banks, voltage regulators and distributed generation or does not consider all these devices in the problem.

Therefore, to ensure the optimal allocation of these devices, this paper uses a quadratic multi objective goal programming method, with the aim of reducing loss, subject to restrictions of power flow, operating limits, change in control of switched capacitor banks and the taps of VRs. The linear power flow used in this paper has the advantage that it does not require an iterative algorithm and the goal programming algorithm seeks a compromise between several conflicting objectives, letting the engineer choose the weight of objectives. In addition, the linear power flow used in this paper has the advantage that it can be used in commercial optimization products, such as Gurobi [22]. The paper is organized as follows: following the introduction, section II describes the methodologies. The study case and the results obtained are presented in section III.

II. METHODOLOGY

A. Linear Power Flow

A linear power flow developed in [16] is used; as a result, the problem does not require an iterative algorithm for the solution. Another advantage of this method is the ability to isolate the real and imaginary parts of voltage variables. In this power flow formulation, the loads are modeled as a combination of constant impedance and constant current accordingly with (1) and (2):

$$\frac{P(V)}{P_0} = C_Z\left(\frac{V}{V_0}\right)^2 + C_I\left(\frac{V}{V_0}\right) \quad (1)$$

$$\frac{Q(V)}{Q_0} = C_Z'\left(\frac{V}{V_0}\right)^2 + C_I'\left(\frac{V}{V_0}\right) \quad (2)$$

The power flow equations using the nodal analysis and the separation of the real and imaginary parts are given by (3) and (4):

$$\sum_{k=2}^{n}(\bar{G}_{m,k}V_k^{re} - \bar{B}_{m,k}V_k^{im}) = I_{p,m} \quad (3)$$

$$\sum_{k=2}^{n}(\bar{G}_{m,k}V_k^{im} - \bar{B}_{m,k}V_k^{re}) = I_{q,m} \quad (4)$$

The real power losses are given by (5):

$$P_{loss} = \sum_{\substack{m,k \\ m<k}}\left[G_{m,k}\right]\left[\left(V_m^{re} - V_k^{re}\right)^2 + \left(V_m^{im} - V_k^{im}\right)^2\right] \quad (5)$$

B. Goal Programming

Goal programming is a method for multi objective analysis, which does not seek to optimize a single objective, but rather, a compromise between several conflicting objectives [17]. The formulation proposed in this study is represented by (6).

$$\begin{aligned} &Min\ \delta_1 + \delta_2 \\ &s.a.\ f_1 - \delta_1 = g_1, \quad f_2 - \delta_2 = g_2 \\ &\delta_1, \delta_2 \geq 0, C_r \end{aligned} \quad (6)$$

Where f_1 and f_2 are the achievement function of the first goal (energy losses) and second goal (total investment cost) respectively; g_1, g_2 are the goals; δ_1, δ_2 are the deviations and C_r is the problem constraints.

The function f_1 evaluates the cost of energy loss of the system ($/year), represented by (7):

$$f_1 = K_e \times \sum_{i=1}^{NLL} P_{loss}(\tau)\tau \quad (7)$$

Where K_e is the energy marginal cost factor in ($/kWh) and NLL is the number of load levels. In this paper, $K_e = 0,3$ and three different load levels are considered.

The function f_2 quantifies the total investment ($/year). It represents the cost of acquisition, maintenance and operation of capacitor banks and VRs.

$$f_2 = \sum n_{SC} \cdot C_{SC} + n_{FC} \cdot C_{FC} + n_{VR} \cdot C_{VR} \quad (8)$$

Where C_{SC}, C_{FC} and C_{VR} represent the costs of switched capacitors banks, fixed capacitor banks and voltage regulators, respectively; and n_{sc}, n_{fc} and n_{vr} indicates the number of switched capacitors banks, fixed capacitor banks and voltage regulators, respectively.

The g_1 and g_2 goals are adjusted by the user, generally is a desired number or a function that describes each objective. In this paper, the goal g_1 is the system energy loss expected to be achieved in each study case (set in $/year), and the goal g_2 is the monetary value to be spent (set in $/year).

Therefore, the difference between the actual state of the system, evaluated in f_1 and f_2, are compared to the goals g_1 and g_2, resulting in the deviations δ_1 and δ_2. These deviations that we want to minimize in the optimization problem.

Equipment costs are show in Table II.

Table II. Equipment costs.

Equipment	Cost ($)
CB F 300 kVA	10.000,00
CB F 600 kVA	12.000,00
CB F 900 kVA	15.000,00
Switched CB	23.000,00
Voltage Regulator	25.000,00

C. Capacitor bank model

Capacitor banks can be fixed (single stage) or have more than one stage (switched). In the latter, due to load variation, the number of units online can be changed for the appropriate voltage adjustment and to reduce loss. Each stage in this work is represented by the integer variable [16]. The inclusion or not of each unit, as well as its level, directly affects the admittance matrix of the system $Y_{m,k}$, multiplying the current stage h_m by the amount of injected reactive $Q_{C,m}$, as shown equation (9):

$$Y_{m,k} = \begin{cases} -y_{m,k} & m \neq k \\ \sum\limits_{\substack{k=2 \\ k\neq m}}^{n} y_{m,k} + jh_m Q_{C,m} + Y_{L,m} & m = k \end{cases} \quad (9)$$

A limit restriction indicating the number of stages available for each capacitor bank should be added and is represented by (10):

$$h_m \le N_{C,m}^{\max} \tag{10}$$

Where $N_{C,m}^{\max}$ is the maximum number of stages available of the capacitor bank connected at node m.

A test in the h_m variable is added to discover if the capacitor is a switch or fixed bank. If the h_m stage is equal in all three load levels considered, than the capacitor bank is fixed. Otherwise, the capacitor bank is a switched one.

D. Voltage regulator model

The transformers in a substation or voltage regulators along the feeder are usually equipped with Under Load Tap Changer capability (ULTC), allowing for voltage adjustment. The voltage adjustment between system buses can also reduce loss. Tap positions are considered as discrete variables. The linear model of voltage regulators described in [16] is used in this study to represent the ULTC capability. This model is shown in Fig. 1, where the variable a represents the transformer ratio.

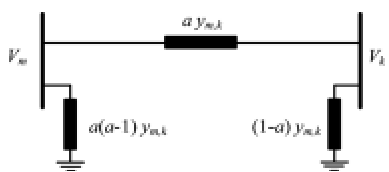

Fig. 1. Linear model of voltage regulator.

The tap ratio usually ranges from 0.9 to 1.1, allowing for the linearization of the variable around the value 1, in 32 steps of 0.00625 p.u..

The only parameters modified by the inclusion of the regulator are the admittances $\bar{G}$ and $\bar{B}$ represented in (3) and (4).

In this study the tap position will be represented by (11):

$$a_m = 1 + rt_m \times (b) \tag{11}$$

Where rt_m is a binary variable that indicates the presence of VR or not in bus m; and *b* represents the 32 allowed positions for the tap.

The auxiliary variable b contains all the 32 steps as show in (12):

$$b = c_1 \times step_1 + c_2 \times step_2 + \ldots + c_n \times step_{32} \tag{12}$$

Subject a:

$$c_1 + c_2 + \ldots + c_n \le 1 \tag{13}$$

The auxiliary variable *c* ensures that only one voltage regulator step is choosed in each bus, and the variable b calculates the respective voltage step in p.u..

Therefore, if the -8 step is chose (c_8=1) in the node 5 ($rt_5 = 1$), the equivalent step in p.u. is -0.05 (b=-0.05 p.u.) and the tap position for the voltage regulator in the node 5 will be 0.95 p.u.

E. Load Levels

The load variation in the literature is often represented by three levels [12], [20] and [21], as in this study. Usually these levels are defined as: light load, medium and heavy load. In this study, the light load level is considered as 70% of the medium level, with a daily duration of six hours, while the heavy load level represents a 20% increase in the medium load, also representing six hours of the day. The medium level has a duration of twelve hours daily.

Table III summarizes these data.

Table III. Load Levels

Load Level	P (kW)	Q (VAR)	Day Hours
Light	2600.5	1610	6
Medium	3715	2300	12
Heavy	4458	2760	6

F. Distributed Generation

The distributed generators are usually operated with a unity power factor to minimize its electrical loss and avoid any charge for consumption of reactive power, regardless of the distribution network needs. Also, in the literature DG has been modeled in power flow studies as a negative load, i.e., as a PQ node [24]. Therefore, in this study, DG is considered as a -PQ constant load in the linear power flow, i.e. the active power and the power factor are known. This model is suitable for small generations which the distributed generation does not have the capacity to control the voltage bus (the opposite of PV approach) [24].

G. Operational Constraints

The upper and lower voltage limits for each bus considered in this paper should be between $V_{\min}^{re} = 0.93$ and $V_{\max}^{re} = 1.05$ representing the real part of the voltage and between $V_{\min}^{im} = -0.1$ and $V_{\max}^{im} = 0.1$, representing the imaginary part. Therefore:

$$V_{\text{re,min}}^{m} \le V_{\text{re}}^{m} \le V_{\text{re,max}}^{m} \tag{14}$$

$$V_{\text{im,min}}^{m} \le V_{im}^{m} \le V_{\text{im,max}}^{m} \tag{15}$$

III. Test System

The IEEE 33 node system described in [18] and shown in Fig. 2 was selected to evaluate the proposal method. The power flow solutions were obtained, assuming as values for the load model, Cz=Cz'= 0.5 and CI = CI' = 0.5 as suggested in [16].

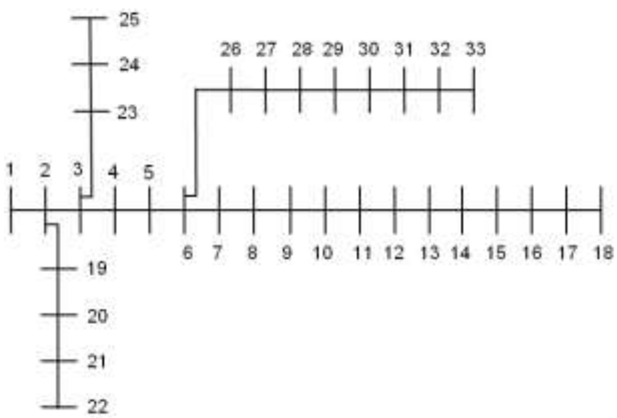

Fig. 2. IEEE 33-node system.

Table IV shows the loads considering the medium level at each node.

Table IV. Loads at medium level.

Node	P (kW)	Q (kVAR)
1	0	0
2	100.00	60.00
3	90.00	40.00
4	120.00	80.00
5	60.00	30.00
6	60.00	20.00
7	200.00	100.00
8	200.00	100.00
9	60.00	20.00
10	60.00	20.00
11	45.00	30.00
12	60.00	35.00
13	60.00	35.00
14	120.00	80.00
15	60.00	10.00
16	60.00	20.00
17	60.00	20.00
18	90.00	40.00
19	90.00	40.00
20	90.00	40.00
21	90.00	40.00
22	90.00	40.00
23	90.00	50.00
24	420.00	200.00
25	420.00	200.00
26	60.00	25.00
27	60.00	25.00
28	60.00	20.00
29	120.00	10.00
30	200.00	600.00
31	150.00	70.00
32	210.00	100.00
33	60.00	40.00

The system total load in this level is 3715 kW and 2300 kVAR.

The final model optimization in GAMS [23] language was implemented in Matlab, containing the objective function and all constraints of the problem. The solver used in all cases was the Knitro being one of the most appropriate solvers for solving mixed-integer quadratic programming problems.

A. Cases

The first case is the named the *basic case*, and it does not consider the presence of capacitor banks, VRs or DG. In the second case, only capacitor banks were added. In the third case, only the voltage regulators were considered. In the fourth case, one DG was added at each time, with 370, 740 and 1110 kW and a unity power factor at buses 10, 16 and 28. In the fifth case, the capacitor banks with the DG of 1110 kW at bus 28 were combined. In the sixth case, the voltage regulators with the DG of 1110 kW at bus 28 were combined. In the last case all equipment are considered. Table V shows the values for the system energy loss goal (g_1) and equipment costs goal (g_2) for all the cases.

Table V. System energy loss and equipment costs goals.

Case	g1 ($/year)	g2 ($/year)
1	-	-
2	400.000	50.000
3	450.000	80.000
4	-	-
5	400.000	30.000
6	400.000	50.000
7	400.000	35.000

A.1. Basic Case

Fig. 3 shows the feeder voltage profile for the basic case for the three load levels.

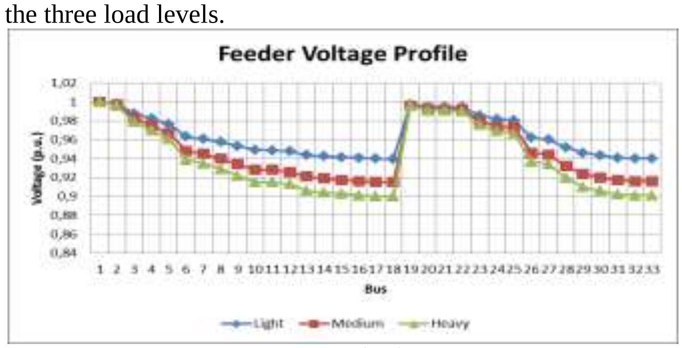

Fig. 3. Feeder voltage profile for the basic case.

Analyzing the feeder voltage level, the lowest voltage modules take place in the heavy load level. The same occurs with the network losses as show in Table VI. The lowest voltage values are 0.9 pu (heavy load), 0.915 (medium load) and 0.9396 pu (light load) in section 18 of the feeder. The light load level is the only one where there is no violation of the minimum voltage limit on the buses, set at 0.93 pu.

Table VI. Power loss for the basic case.

Load Level	Power Loss (kW)
Light	102.8
Medium	202.4
Heavy	278.82

A.2 Cases 2, 3 and 4.

Table VII summarizes the cases considered with respective values of loss and reduction compared to the base case. For the fourth case, only the result of DG at bus 28 with 1110 kW is show.

Table VII. Energy loss for cases 1 to 4.

Case	Energy Loss ($/year)	Equipment Costs ($/year)	Reduction (%)
1	510.632,33	0,00	-
2	366.210,29	43000.00	19.86
3	425.378,33	75000.00	2.01
4 DG (28)	336.315,02	-	34.14

Table VIII shows the results of capacitor placement for the second case, while Table XIX shows the solution for voltage regulators for the third case.

Table VIII. Capacitor placement for the second case.

Load Level	Capacitor (Bus)	Type of the bank	Size (kVAR)
	2	Fixed	300
Light	12	Fixed	300
	30	Switched	300
	2	Fixed	300
Medium	12	Fixed	300
	30	Switched	600
	2	Fixed	300
Heavy	12	Fixed	300
	30	Switched	900

Table XIX. Voltage regulator placement solution for the third case.

Load Level	VR placement	TAP
	6	0.99375
Light	17	0.99375
	30	1.00625
	6	0.9875
Load	17	0.99375
	30	1.00625
	6	0.98125
Heavy	17	0.9875
	30	1.01875

Table X shows the results of DG placed at bus 28 with 370, 740 and 1110 kW for the fourth case.

Table X. Energy Loss for DG at bus 28.

DG Power (kW)	Energy Loss ($/year)
370	423.054,13
740	368.741,22
1110	336.315,02

Fig. 4 shows the feeder voltage profile for the second case (only CBs) for the three load levels.

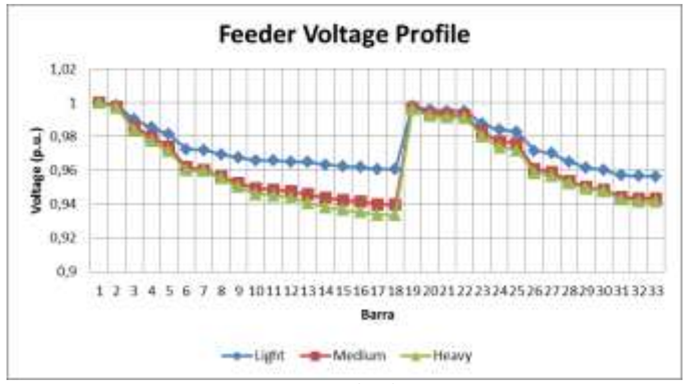

Fig. 4. Feeder voltage profile for the second case.

A.3 All cases

Table XI summarizes the best scenarios for all cases considered and Fig. 5 shows the feeder voltage profile for the seventh case.

Table XI. Energy loss for 33-node system: all cases.

Case	Energy Loss ($/year)	Equipment Costs ($/year)	Reduction (%)
1	510.632,33	0,00	-
2	366.210,29	43000,00	19.86
3	425.378,33	75000,00	2.01
4 DG (28)	336.315,02	-	34.14
5 DG (28)	174.238,89	20000,00	61.96
6 DG (28)	296.211,61	25000,00	37.10
7 DG (28)	174.238,89	20000,00	61.96

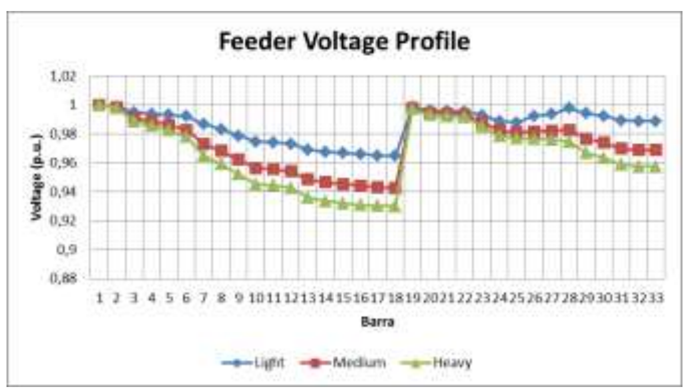

Fig. 5. Feeder voltage profile for the seventh case.

IV. Conclusion

In this paper a goal programing methodology has been proposed for optimal placement of capacitor banks and voltage regulators considering the presence of distributed generation in the system. The models have been developed with the aim of quantifying the effect of each device on the electrical loss in the system and its investment costs.

Based on the results of the IEEE 33 node, it was observed that this system has two sections with a critical voltage level; therefore, the algorithm tends to allocate at least one piece of equipment to these sections (the most sensitive buses), to meet the constraint of minimum voltage, especially in heavy load periods.

Comparing cases 2 and 3, it was observed that the system with capacitor banks has lower values of energy loss and costs when compared to voltage regulators. In case 4 with only distributed generation, the minimum voltage restriction in the buses did not occur in any example. Therefore, a high value of penetration was needed to meet this requirement (penetration higher than 30%), which could result in a voltage value above the established upper limit (1.05 pu) in the light period load.

In addition, the fourth case shows that the distributed generation has a greater effect on the system loss as the penetration is increased. When DG was allied with the capacitor banks (case five), the highest percentage of cost reduction occurred, representing a decrease of 61.96% for a DG of 1110 kW installed on bus 28 (higher DG penetration). For the sixth case, when the same DG of the five case, and the voltage regulators are considered, a reduction of 37.10% is presented. Cases 7 demonstrated that the voltage regulator is not a good financial solution when it is compared to capacitor banks and DG, therefore only capacitor banks are selected in this case.

Analyzing the feeder voltage profile show in Figs. 3, 4 and 5, the three capacitor banks installed in the case 2 improves more the system voltage profile, especially in the heavy load, than the case seventh (one capacitor bank and one DG), with a minimum voltage of 0.9335 p.u. while in the case 7 the minimum voltage was 0.93 p.u..

Due to increase in DG penetration in distribution systems, its impact in system operation and costs cannot be disregarded In addition, the results shows that this type of generation can be used in the PDS, replacing the traditional solutions that are commonly used to minimize system loss.

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