# Forecasting Power Quality Events Using Advanced Fuzzy Time Series

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***Abstract*—This paper proposes the study of advanced fuzzy time series to forecast power quality events for an industrial power system. Power quality events predictions makes significantly influence to the availability and reliability of an electrical system. The standards reliability models usually used to predict stochastically these events are mathematical and statistical complex and the use of fuzzy time series brings new fields to reduce this complexity and time processing.**

***Index Terms*—Power Quality Events, Power Quality Indices, Fuzzy Time Series, Forecasting Models, Reliability Theory.**

## I. Introduction

The continuous growing of power system in the world brings lots of challenges in the fields of power systems stability, power quality, system protections and other fields. To deal with this challenge, some reliability evaluation methods are developed to study and predict power quality events. The reliability evaluation methods are today's state of the art to predict power quality events [1] [2] [3].

The use of fuzzy time series for solving problems which data can be interpreted as vague, linguistic, fuzzy values or numbers was introduced first by Song and Chissom in 1993 [4] [5] [6] [7]. The advantages of using fuzzy time series lies in the possibility to be implemented by algorithms, are easy to improve algorithms to increase forecast accuracy and no need to have strong background in mathematics and statistics.

In this paper, the set of lifetime data are the stochastic power quality events, primarily interruptions and voltage sags, that makes significantly influence to the availability and reliability of an electrical system. These parameters are forecasting using the fuzzy time series and then compared with state of the art reliability models usually used to predict them. The results obtained in this work shows that the use of advanced fuzzy time series reduces the mathematical and statistics complexity of the models, brings less machine time consumption and better forecast accuracy than standard reliability models.

## II. Power Quality Events and Indices

Disturbances in the electrical network caused primarily by interruptions and voltage sags represent the vast majority of power quality problems faced by typical industrial consumers who increasingly use sophisticated equipment in their control processes. Interruptions in this processes significantly impact the economy due to the high profit loss typical from manufacturing base industries. According to EPRI (Electric Power Research Institute), power-related problems cost U.S. companies \$26 billion dollars a year in lost time and revenue [8] [1].

The stochastic nature of power quality events significantly influence the availability of an electrical system. The importance of knowing this availability has motivated several studies of reliability, since consumers, mainly commercial and industrial sectors are increasingly sensitive to profit losses associated with unavailability of electrical system. Interruptions and voltage sags are stochastic events caused primarily by equipment failures, power system faults and others problems related to control system.

Interruptions are by far the most severe power quality event and is defined as voltage or current drop below 0.1 p.u. with duration longer than 1 minute [8]. SAIFI (System Average Interruption Frequency Index):

$$\overline{\lambda} = \frac{\sum_{i=1}^{k} N_i}{N_T} \tag{1}$$

and SAIDI (System Average Interruption Duration Index):

$$\overline{q} = \frac{\sum_{i=1}^{k} N_i D_i}{N_T} \tag{2}$$

where:

| | |
|---|---|
| $\overline{\lambda}$ : | system average interruption frequency index; |
| $\overline{q}$ : | system average interruption duration index; |
| $k$: | total number of power system outages during a year; |
| $N_i$: | number of customers affected by $i$-th outage; |
| $D_i$: | duration, in minutes, of $i$-th outage; |
| $N_T$: | total number of customers. |

are the most commonly interruptions indices reported by electricity industry. Voltage sags are short duration reductions in RMS voltage, caused by short circuits, overloads and starting of large motors [1]. This event is characterized by a voltage drop between 0.9 and 0.1 p.u. with duration between half-cycle and 1 minute [8].

Various methods are available to estimate and predict power quality events. The basic ones are the power quality indices described by (1), (2) and IEEE Std-1366 indices [9]. An alternative and more accurate method to predict power quality events is the use of probability distributions for the lifetime of power quality events ($T_E$):

$$T_E = [T_{E,1}, T_{E,2}, ..., T_{E,n}] \tag{3}$$

Time series are also an alternative method and in this paper it will be studied the use of fuzzy time series with reliability evaluation to better predict these events.

## III. Reliability Theory

Life data analysis is a field of reliability engineering concerned to determine the probability of failures for a given set of failure data. Life data can be lifetimes of equipments or products in the marketplace and can be expressed as the time equipment/product operated successfully or the time that equipment/product operated before it failed. These lifetimes can be measured in days, hours, miles, cycles-to-failure or any other metric with which the life or exposure of a product can be measured [10].

The basis of life data analysis is supported in the study and knowledge of the statistical distribution and its parameters. There are several distributions functions used in life data analysis and reliability engineering, the most used are exponential, normal, lognormal and Weibull. The probability density function of the Weibull distribution (4) is one of the most used in reliability engineering. The reason for that is due to the versatility of this distribution in relation to the others distributions such as exponential, normal and lognormal [10].

The distribution parameters of Weibull probability density, cumulative distribution and reliability functions are respectively given by (4) and (5):

$$f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} \tag{4}$$

$$F(t) = 1 - R(t) = 1 - e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} \tag{5}$$

where:

| | |
|---|---|
| $f(t)$: | probability distribution function; |
| $F(t)$: | cumulative distribution function; |
| $R(t)$: | reliability function; |
| $t$: | time; |
| $\beta$: | shape parameter; |
| $\eta$: | scale parameter; |
| $\gamma$: | minimum life. |

can bring certain conditions of equipment/process obtained by correlation with the life data from this equipment/process. The Weibull shape parameter are correlated with the various stages of equipment life, i.e, early life ($\beta < 1$), useful life ($\beta = 1$) and Wear-out life ($\beta > 1$). The scale parameter provides information on the occurrence of a failure when it happen with probability of 63.2%.

To obtain the parameters for a given distribution and a set of data, the maximum likelihood method:

$$\Lambda = \ln(L) = \sum_{i=1}^{R} \ln f(x_i; \theta_1, \theta_2, ..., \theta_k) \tag{6}$$

from the statistical point of view, is considered the most robust method to estimate these parameters. The main idea about this method stands in determining the most probable parameters values of functions that fits a given set of data [10] [11] [12]. The parameters are obtained when the maximum value for $\Lambda$ is reached using some class optimizations algorithms like Nelder-Mead, Ellipsoidal and others [13] [14] [15].

## IV. Fuzzy Time Series

Time series is a set of data indexed in time order or a collection of observations made sequentially in time [16]. Most commonly, a time series is a sequence taken at successive equally spaced points in time. Thus it is a sequence of discrete-time data. Examples of time series are the mean daily temperature of a city, crude oil price per barrel and so on. Time series analysis comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data. Time series forecasting is the use of a model to predict future values based on previously observed values [17]. Time series could be divide in two types, i.e., i-Deterministic Time Series, where the pattern of the component of time series almost fixed as time passed, and ii-Stochastic Time Series where the pattern of the component of time series changing during time [16]. Many methods to predict time series were developed during decades, the main methods are the Naive model, Mean method, Moving Average Smoothing, Weighted Mean method, Exponential Smoothing, Additive Holt-Winters method, ARMA and ARIMA models.

Fuzzy time series (FTS) combines the fuzzy set theory developed by Zadeh in 1965 with time series analysis. According [18], the fuzzy set theory is being applied into wider and wider areas, such as decision making, planning, logic, systems theory, artificial intelligence, economics, control theory and so on. The main reason for using fuzzy sets instead of crisp sets is the advantage to deal with formal, powerful and quantitative framework to cope with the vagueness of human knowledge as it is expressed by means of natural languages. The methodology around FTS are discussed briefly during the explanation of these FTS methods described in items from IV-A to IV-D. The results obtained will be discussed in details on section V.

### A. Yu FTS Algorithm

The Yu FTS algorithm or Weighted FTS is very similar to the Chen FTS algorithm [16] [19]. A complete description of this algorithm could be viewed in [20]. The steps to implement the algorithm are presented below.

***Step 1***. Define the universe of discourse:

$$U = \left[D_{min} - D_1, D_{max} + D_2\right] \tag{7}$$

based on historical data, min and max values of the data set and proper numbers $D_1$ and $D_2$ to adjust the range of data. Another way to define the universe of discourse could be implement using $k$-means clustering as presented in [21].

***Step 2***. Partition the universe of discourse (7) into $n$-intervals with equal length:

$$U = \left[u_1, u_2, ..., u_n\right] \tag{8}$$

The number of intervals could be defined by experience of the analyst or by some analytical, e.g., Huarng method or evolutionary algorithms that find the optimum number of intervals [16].

***Step 3***. Define fuzzy sets on the universe $U$. The fuzzy sets:

$$A_i = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_2 + ... + f_{Ai}(u_b)/u_b \tag{9}$$

could be defined using some linguistic values such as $A_1$ = (not many), $A_2$ = (not too many), $A_3$ = (many), $A_4$ = (many many), $A_5$ = (very many), $A_6$ = (too many), $A_7$ = (too many many). In this paper the fuzzy sets are defined according to:

$$\begin{aligned} &A_1 = \{u_1/1, u_2/0.5, ..., u_{n-1}/0, u_n/0\} \\ &A_2 = \{u_1/0.5, u_2/1, ..., u_{n-1}/0, u_n/0\} \\ &\vdots \\ &A_n = \{u_1/0, u_2/0, ..., u_{n-1}/0.5, u_n/1\} \end{aligned} \tag{10}$$

***Step 4***. Fuzzify historical data. In some FTS methods the assign values of the fuzzy sets is subjective. In this paper a modular function was adopted as presented in:

$$f_A(m,n) = \begin{cases} 1.0, & if\ D(m) \geq U(1,n) \wedge \\ & D(m) \leq U(2,n); \\ 0.9, & if\ (\Delta > 0.50\Delta u) \wedge \\ & (\Delta \leq 0.75\Delta u); \\ 0.8, & if\ (\Delta > 0.75\Delta u) \wedge \\ & (\Delta \leq 1.00\Delta u); \\ 0.7, & if\ (\Delta > 1.00\Delta u) \wedge \\ & (\Delta \leq 1.25\Delta u); \\ 0.6, & if\ (\Delta > 1.25\Delta u) \wedge \\ & (\Delta \leq 1.50\Delta u); \\ 0.5, & if\ (\Delta > 1.50\Delta u) \wedge \\ & (\Delta \leq 1.75\Delta u); \\ 0.4, & if\ (\Delta > 1.75\Delta u) \wedge \\ & (\Delta \leq 2.00\Delta u); \\ 0.3, & if\ (\Delta > 2.00\Delta u) \wedge \\ & (\Delta \leq 2.25\Delta u); \\ 0.2, & if\ (\Delta > 2.25\Delta u) \wedge \\ & (\Delta \leq 2.50\Delta u); \\ 0.1, & if\ (\Delta > 2.50\Delta u) \wedge \\ & (\Delta \leq 2.75\Delta u); \\ 0.0, & if\ (\Delta > 2.75\Delta u). \end{cases} \tag{11}$$

to automate this process. This function uses the $n$-interval length of $U$:

$$\Delta u = U(2,1) - U(1,1) \tag{12}$$

and the distance between the $m$-actual data, $D(m)$, with respect to the mean of the $n$-interval length.

$$\Delta = \left| D(m) - \frac{\Delta u}{2} \right| \tag{13}$$

***Step 5***. Establishing fuzzy logical relationships (FLR). This process find the sequence of fuzzy sets that is equal to 1, i.e., $A_i$ = 1:

$$\begin{aligned} &A_1 \to A_1 \\ &A_1 \to A_1 \\ &\vdots \\ &A_6 \to A_6 \\ &A_6 \to A_7 \end{aligned} \tag{14}$$

***Step 6***. Forecast all the right hand side of the fuzzy data in FLR according to:

$$F(i) = [M_1, M_2, ..., M_i] \times [w_1, w_2, ..., w_i]^T \tag{15}$$

with the weighted matrix defined by (16), (17) and (18).

$$W(t) = [w_1^{'}, w_2^{'}, ..., w_k^{'}] \tag{16}$$

$$W(t) = \left[ \frac{w_1}{\sum\limits_{h=1}^{k} w_h}, \frac{w_2}{\sum\limits_{h=1}^{k} w_h}, ..., \frac{w_k}{\sum\limits_{h=1}^{k} w_h} \right] \tag{17}$$

$$\sum_{h=1}^{k} w_h^{'} = 1 \tag{18}$$

where:

| | |
|---|---|
| $i = k$: | total number of fuzzy sets in the right hand side of FLR; |
| $M$: | midpoint of the $n$-intervals in the universe of discourse $U$; |
| $W$: | calculated weights for each midpoint Mi's. |

## *B. Exponential FTS Algorithm*

The Exponential FTS algorithm makes some corrections in the weights calculated in Yu FTS algorithm (17) and (18). The steps to implement the exponential FTS are the same that Yu FTS with some differences when calculate the weights (19). The adjustment of parameter $c$ makes great difference in fuzzy forecast, usually the value $c$ is set as 1.2.

***Step 1***. Implement the steps from 1 to 5 according Yu FTS algorithm.

***Step 2***. Forecast all the right hand side of the fuzzy data in FLR according to (15) with the weighted matrix defined by (16) and:

$$W(t) = \left[ \frac{1}{\sum\limits_{h=1}^{k} w_h}, \frac{c}{\sum\limits_{h=1}^{k} w_h}, ..., \frac{c^2}{\sum\limits_{h=1}^{k} w_h}, \frac{c^{k-1}}{\sum\limits_{h=1}^{k} w_h} \right] \tag{19}$$

where:

$i = k$: total number of fuzzy sets in the right hand side of FLR;
$c$: weight constant with $c \geq 1.2 \leq i \leq k$;
$M$: midpoint of the $n$-intervals in the universe of discourse $U$;
$W$: calculated weights for each midpoint Mi's.

### C. Transformation FTS Algorithm

Transformation FTS algorithm is commonly used to remove noisy effects of data and therefore increases the forecastability of time series [16]. The Box Cox transformation is applied to the actual data series. The steps to implement the Transformation FTS algorithm are presented as follow:

***Step 1***. Apply Box Cox transformation to the actual data series and then obtain the $\lambda$ parameter and $Z$ vector:

$$Z_t^\lambda = \begin{cases} \frac{Z_t^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \\ \ln(Z_t) & \lambda = 0 \end{cases} \tag{20}$$

***Step 2***. Calculate $d_S$ vector according to:

$$d_s = \frac{Z_S^\lambda - Z_{S-1}^\lambda}{Z_S^\lambda} \tag{21}$$

***Step 3***. Calculate $\Delta_q$ as a difference of the third and first quartile, according to:

$$\Delta_q = Q_3 - Q_1 \tag{22}$$

***Step 4***. Calculate parameter $l$ according to:

$$l = \frac{2 \times \Delta_q}{n^{1/3}} \tag{23}$$

***Step 5***. Obtain the estimated vector $d_S$ using Yu FTS algorithm.

$$\hat{d}_S = [M_1, M_2, ..., M_i] \times [w_1, w_2, ..., w_i]^T \tag{24}$$

***Step 6***. Calculate the estimated $y_S$ parameter according to:

$$\hat{y}_S = \frac{1}{1 - \hat{d}_S} \tag{25}$$

***Step 7***. Apply:

$$\hat{Z}_S^\lambda = \hat{y}_S Z_t^\lambda \tag{26}$$

to the vector $Z$ obtained in (20).

***Step 8***. Obtain the forecast data series according to:

$$F = \hat{Z}_S = \left(\lambda \hat{Z}_S^\lambda + 1\right)^{1/\lambda} \tag{27}$$

### D. High-Order FTS Algorithm

In some cases the data to be forecast are not related to the last state, thus they are related to some sequence of former states. According [16] the sequence of former states can be done by some of these options below:

- by expert knowledge;
- by data discovery;
- by applying try and error procedure;
- by ACF (Auto-correlation function) analyzing;
- by some evolutionary algorithm.

High-Order FTS are very usefull for seasonal and multiseasonal series. The steps to implement High-Order FTS algorithm are related below:

***Step 1***. Implement the steps from 1 to 4 according Yu FTS algorithm.

***Step 2***. Choose the sequence of former states according strategies presented before. For example:

$$F(t-2), F(t-1) \to F(t) \tag{28}$$

presents the states according example in [16].

***Step 3***. Establishing fuzzy logical relationships (FLR) according the former states. In:

$$\begin{aligned} &A_1, A_1 \to A_1 \\ &A_1, A_1 \to A_2 \\ &\vdots \\ &A_6, A_7 \to A_7 \\ &A_7, A_7 \to A_6 \end{aligned} \tag{29}$$

the FLR are composed of two former states [16].

***Step 4***. Eliminate the recurrence of fuzzy logical relationship (29) and defines the fuzzy logical relationship group (FLRG) for the former states:

$$\begin{aligned} &A_1, A_1 \to A_1, A_2 \\ &A_1, A_2 \to A_3 \\ &\vdots \\ &A_6, A_6 \to A_7 \\ &A_7, A_7 \to A_6 \end{aligned} \tag{30}$$

***Step 5***. Forecast all the right hand side of the fuzzy data in FLR according to:

$$F(i) = \frac{\sum_{j=1}^{i} M_j}{i} \tag{31}$$

## V. Results

The power quality data events analyzed by the FTS algorithms was obtained from measurements realized in a power control room from a non-ferrous metals industry located at Minas Gerais - Brazil. The electric power system from this industry are made of 1 transmission line of 138 kV with 12 towers, 1 main substation of 138 kV and 36 substations of 13.8 kV. The total power installed are 105 MW.

The data recorded for power quality events has sampled 7 points for interruptions and 67 for voltages sags, and was acquired during period from jan-2005 to jun-2008. The life data analysis uses maximum likelihood method and Weibull distribution function to estimate parameters for interruption ($\beta$ = 1.07, $\eta$ = 5486) and voltages sags ($\beta$ = 0.47, $\eta$ = 288), these distributions were used to generate 30 points for the time series. Figures 1 and 2 presents the best fuzzy time series for forecasting interruption and voltage sag events. All these figures uses blue dots for the generated sample and red dots for FTS forecast. Figure 3 and 4 compares best FTS forecast with reliability evaluation method for predict power quality events.

The evaluation methods for fuzzy time series applied where obtained using forecast errors measures, i.e, MAPE - Mean Absolute Percentage Error (referred as mean error), MAD - Mean Absolute Deviation, MSE - Mean Squared Error and RMSE - Root Mean Squared Error. For all these methods the smaller value are the better method that fits the model.

### A. Yu FTS Algorithm

The mean error for forecast interruption events using Yu FTS algorithm was 30.35% and 1370% for forecast voltage sag events.

### B. Exponential FTS Algorithm

For Exponential FTS algorithm the mean error was 30.35% for forecast interruption events. For voltage sag events the mean error was 1370%.

### C. Transformation FTS Algorithm

The Transformation FTS algorithm implemented obtain a mean error of 8.23% for forecast interruption events (Figure 1) and 2.27% for forecast voltage sag events (Figure 2). Figures 3 and 4 presents the Transformation FTS forecast and PoF (Probability of Failure) comparisons for power quality events.

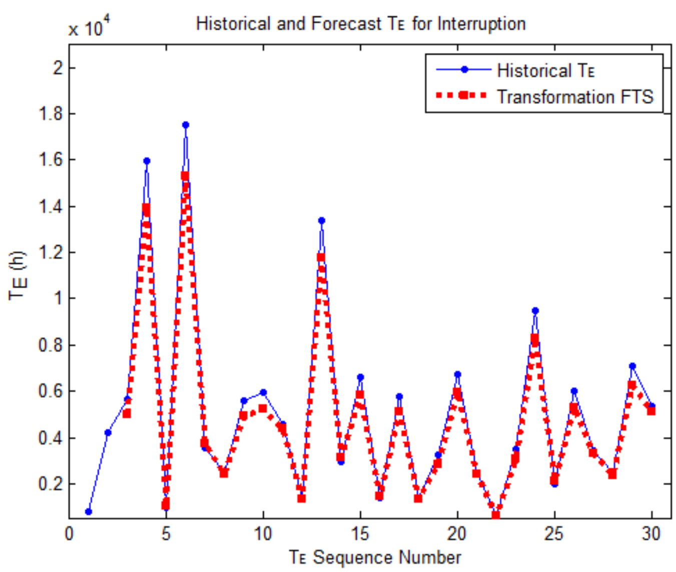


Fig. 1. Transformation FTS forecast for Interruption.

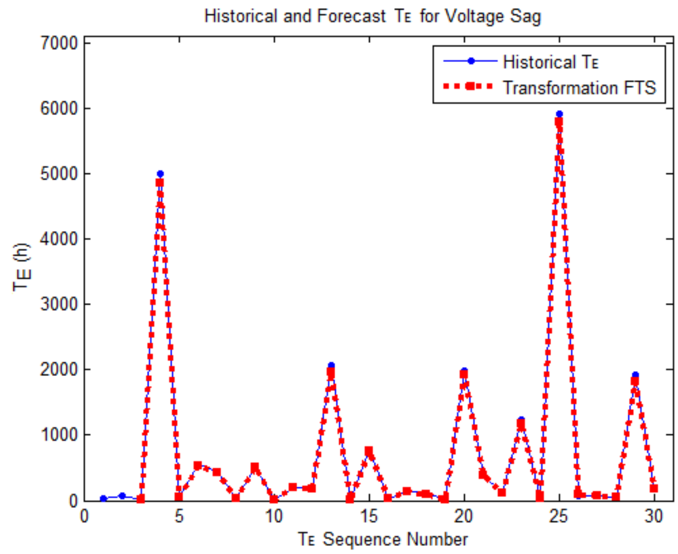


Fig. 2. Transformation FTS forecast for Voltage Sag.

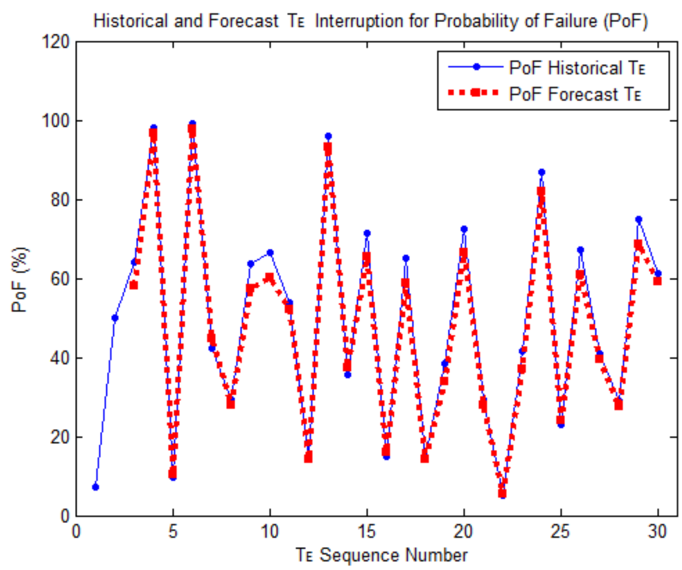


Fig. 3. FTS forecast and PoF comparisons for Interruption.

### D. High-Order FTS Algorithm

In High-Order FTS algorithm, mean error was 54.65% for forecast interruption events and 3470% for forecast voltage sag events. The sequence of former states used in this algorithm was chosen according to:

$$F(t-3), F(t-2), F(t-1) \rightarrow F(t) \tag{32}$$

### E. Summary of Statistics Estimated Errors

Table I and II presents comparisons between fuzzy forecast methods and power quality events according four different error measures methods.

## VI. Conclusion

From the results obtained in section V it is possible to verify that Transformation FTS algorithm was the best

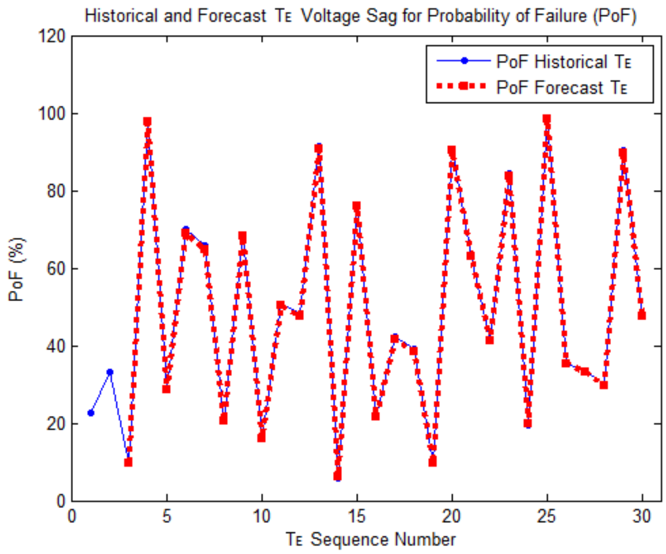


Fig. 4. FTS forecast and PoF comparisons for Voltage Sag.

TABLE I
INTERRUPTION ERROR COMPARISON

| Forecast Method | MAPE | MAD | MSE | RMSE |
|---|---|---|---|---|
| Yu FTS | 30.4 | 804 | $8.06\times10^5$ | 898 |
| Exponential FTS | 30.4 | 804 | $8.06\times10^5$ | 898 |
| Transformation FTS | 8.23 | 545 | $6.39\times10^5$ | 800 |
| High-Order FTS | 54.7 | 1700 | $5.82\times10^6$ | 2410 |

TABLE II
VOLTAGE SAG ERROR COMPARISON

| Forecast Method | MAPE | MAD | MSE | RMSE |
|---|---|---|---|---|
| Yu FTS | 1370 | 247 | $7.11\times10^4$ | 267 |
| Exponential FTS | 1370 | 247 | $7.11\times10^4$ | 267 |
| Transformation FTS | 2.27 | 24.2 | $2.30\times10^3$ | 47.9 |
| High-Order FTS | 3470 | 978 | $2.01\times10^6$ | 1420 |

fuzzy time series method to fit power quality events for the non-ferrous metals industry case. It also can be viewed in Figures 3 and 4 that reliability evaluation fails to predict interruptions and voltage sags when the probability (PoF) are less than 70%. The use of FTS algorithms instead of standard reliability evaluation to predict power quality events has the following advantages:

- the mathematical and statistics background necessary in FTS is much more simple and easy to understand and implement than the standard reliability evaluation prediction methods;
- due to high accuracy in FTS to predict power quality events, such as interruptions and voltage sags, it can be used to manage improvements in electrical power system protections, avoiding profit loss due to the effects of these events.

For future improvements of this work it shall be implement some class of evolutionary algorithms like Genetic Algorithm (GA), Differential Evolution (DE) or Particle Swarm Optimization (PSO) to find the best sequence of former states (28) and to find optimum length of the intervals (8). This improvements could reach better accuracy on the forecasted time series.

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