

IMPROVED ANGULAR POSITION ESTIMATION FOR PMSM BASED ON MODIFIED LUENBERGER OBSERVED AND LINEAR MATRIX INEQUALITIES

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Abstract— This paper describes a simple and accurate angular position estimation algorithm for a permanent magnet synchronous motor which is based on a modification of the Luenberger observer. The Luenberger observer is used to estimate the back-electromotive forces in the stationary $\alpha\beta$ reference systems. The conventional approach considers the derivatives of the back-electromotive forces as negligible. However, this assumption is not true for high mechanical speeds. In this paper, a better space-state model of the motor is proposed, using the derivatives of the back-electromotive forces defined in an average operation point. The proposed state-space model represents more accurately the dynamics of the motor than the conventional model, allowing a better estimation of the back-electromotive forces. Thus, better observer and observation matrices can be obtained. The observation matrix is set through Linear Matrix Inequalities. Simulation results show that the proposed observer is more accurate than the conventional Luenberger observer.

Keywords— Back-electromotive force, Luenberger observer, PMSM, sensorless.

Resumo— Este artigo descreve um algoritmo de estimação de posição angular para um motor síncrono de ímã permanente simples e preciso, o qual está baseado em uma modificação no observador de Luenberger. O observador de Luenberger é usado para estimar as forças contra-eletromotrices no sistema estacionário $\alpha\beta$. A técnica convencional considera que as derivadas das forças contra-eletromotrices são consideradas como desprezíveis. Não obstante, esta consideração não é certa para altas velocidades mecânicas. Neste artigo, foi desenvolvido um melhor modelo em espaço de estado do motor, usando as derivadas das forças contra eletromotrices em um ponto médio de operação. O modelo em espaço estado proposto representa melhor a dinâmica do motor que o modelo convencional. Desta maneira, melhores observadores e matriz de observação são obtidas. A matriz de realimentação é definida através de inequações lineares matriciais. Resultados de simulação mostram que o observador proposto é mais preciso que o observador de Luenberger convencional.

Palavras-chave— Força contra-eletromotriz, observador de Luenberger, MSIP, sistema sem sensores.

1 Introduction

Nowadays, Permanent Magnet Synchronous Motor (PMSM) is a powerful alternative in the development of small and medium variable frequency drives and servomechanisms due to its better relation torque/weight and robustness than DC motors. On the other hand, PMSM has less rotor power losses and less torque ripple than induction motors (Uddin et al., 2000; Rashid, 2001; Terzic and Jadric, 2001).

In order to control a PMSM, it is necessary information about the angular position and speed. Sensors such as encoders and resolvers can be used as position transducer. However, it can increase the cost and reduce the robustness of the motor drive. For that reason, many researches were done to eliminate the need for position sensors in motor drives, i.e., a sensorless control (Teja et al., 2015). These sensorless algorithms are based in techniques such as sliding mode observers (Bernardes et al., 2014; Zhao et al., 2014), Kalman filters (Alonge et al., 2014; Quang et al., 2014), MRAS (Fan et al., 2012), injection of high-frequency signals (Wang et al., 2013; Luo et al., 2016), Luenberger observers (STMICROELECTRONICS, 2007) and others.

Many of these algorithms are precise but have high computational costs. On the other hand, Luenberger observer is simple to implement: the observer is used to estimate the back-electromotive forces in the stationary $\alpha\beta$ reference system. The angular position and mechanical speed are calculated through the estimated back-electromotive forces. However, the conventional approach assumes that the derivative of the back-electromotive force is near to zero (STMICROELECTRONICS, 2007). This assumption is not true for high speeds.

For these reason, this paper presents a modification of the Luenberger observer in order to increase the accuracy of the angle estimation for a PMSM, without increasing significantly the complexity of the observer. This approach considers that the derivatives of the back-electromotive forces in the stationary reference system are not zero. This consideration adds non-linear terms to the state matrix of the observer. However, in order to avoid a non-linear space-state observer, these derivatives are defined in an average operation point. Additionally, it is consider that the acceleration of the motor is small. Thus, the new terms of the state matrix are constant, and the proposed Luenberger observer is still linear.

The observation matrix of the proposed Luenberger matrix is set through Linear Matrix Inequalities (LMIs). The new state matrix of the proposed observer also changes the structure of the observation matrix.

This paper is organized as follows: Section 2 describes the modeling of a PMSM and the Luenberger observer applied to the angle estimation. Section 3 describes the proposed modified Luenberger observer. Section 4 shows simulations results that validate the proposed technique. Finally, conclusions and future works are presented.

2 Theoretical Foundations

2.1 Modeling of a PMSM

The high number of parameters and the time-varying inductances make the three-phase model of a PMSM inadequate for the analysis of this kind of motor. In order to get a better model of a PMSM, Park transformation is used to represent three-phase quantities in the dq orthogonal reference system (Pillay and Krishnan, 1991):

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \sin(\delta) & \sin(\delta-k) & \sin(\delta+k) \\ \cos(\delta) & \cos(\delta-k) & \cos(\delta-k) \\ 0,5 & 0,5 & 0,5 \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad (1)$$

where $k = 2\pi/3$, $[x_a \ x_b \ x_c]^T$ are the vector with the three-phase quantities (voltages, currents or fluxes), $[x_d \ x_q \ x_0]^T$ is the vector with the new dq variables, while δ is the angle of the new reference system. Usually, $\delta = \theta_e = n\theta$, where θ_e is the electrical angle of the PMSM, n the number of pair of poles and θ is the shaft angle. The following equations define the dq model of the PMSM:

$$v_d = ri_d + l_d \frac{di_d}{dt} - nl_q \omega_m i_q \quad (2)$$

$$v_q = ri_q + l_q \frac{di_q}{dt} + nl_d \omega_m i_d + n\phi_m \omega_m \quad (3)$$

$$t_{em} = 1,5n(l_d - l_q)i_d i_q + 1,5n\phi_m i_q \quad (4)$$

$$t_{em} - t_{load} = j_r \frac{d\omega_m}{dt} + b_m \omega_m \quad (5)$$

where v_d and v_q are the dq stator voltages, i_d and i_q are the dq stator currents, r is the stator resistance, l_d and l_q are the dq stator inductances, ϕ_m is the equivalent flux of the magnets, t_{em} is the electromagnetic torque, t_{load} is the load torque, j_r is the rotor inertia, b_m is the coefficient of friction, and ω_m is the mechanical speed (the derivative of θ). The d -axis and q -axis inductances of a PMSM with non-salient poles are equal:

$$l_d = l_q = l \quad (6)$$

Replacing (6) in (4), we get:

$$t_{em} = 1,5n\phi_m i_q \quad (7)$$

However, many speed observers are based on the stationary $\alpha\beta$ model of the PMSM, which is obtained through Park transformation in (1) when $\delta = 0$. In that case, PMSM can be modeled by the following equations:

$$\frac{di_\alpha}{dt} = -\frac{r}{l}i_\alpha + \frac{1}{l}v_\alpha - \frac{1}{l}e_\alpha \quad (8)$$

$$\frac{di_\beta}{dt} = -\frac{r}{l}i_\beta + \frac{1}{l}v_\beta - \frac{1}{l}e_\beta \quad (9)$$

where e_α and e_β are the back-electromotive forces:

$$\begin{aligned} e_\alpha &= n\phi_m \omega_m \cos(n\theta) \\ &= n\phi_m \omega_m \cos(\theta_e) \end{aligned} \quad (10)$$

$$\begin{aligned} e_\beta &= n\phi_m \omega_m \sin(n\theta) \\ &= n\phi_m \omega_m \sin(\theta_e) \end{aligned} \quad (11)$$

2.2 Speed Estimation for a PMSM based on Luenberger Observer

The space-state model of a linear system can be defined as follows:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx} \end{aligned} \quad (12)$$

where $\mathbf{x}$ is the state vector, $\mathbf{u}$ is the input vector, $\mathbf{y}$ is the output, $\mathbf{A}$ is the state matrix, $\mathbf{B}$ is the input matrix and $\mathbf{C}$ is the output matrix. The Luenberger observer has the following structure:

$$\begin{aligned} \dot{\mathbf{x}}_{est} &= \mathbf{Ax}_{est} + \mathbf{Bu} + \mathbf{L}(\mathbf{y} - \mathbf{y}_{est}) \\ \mathbf{y}_{est} &= \mathbf{Cx}_{est} \end{aligned} \quad (13)$$

where $\mathbf{L}$ is the observation matrix, while $\mathbf{x}_{est}$ and $\mathbf{y}_{est}$ are the estimated state vector and output vector, respectively. From (12) and (13), we have:

$$\begin{aligned} \dot{\mathbf{x}} - \dot{\mathbf{x}}_{est} &= \mathbf{A}(\mathbf{x} - \mathbf{x}_{est}) - \mathbf{L}(\mathbf{y} - \mathbf{y}_{est}) \\ &= \mathbf{A}(\mathbf{x} - \mathbf{x}_{est}) - \mathbf{LC}(\mathbf{x} - \mathbf{x}_{est}) \\ &= (\mathbf{A} - \mathbf{LC})(\mathbf{x} - \mathbf{x}_{est}) \end{aligned} \quad (14)$$

Let $\mathbf{z} = \mathbf{x} - \mathbf{x}_{est}$ be the error vector. Thus, (14) can be rewritten as follows:

$$\dot{\mathbf{z}} = (\mathbf{A} - \mathbf{LC})\mathbf{z} \quad (15)$$

Luenberger observer can be used to estimate the electrical angle of a PMSM, based on the estimation of the back-electromotive force. Typically, it is assumed that the derivatives of e_α and e_β are zero:

$$\frac{de_\alpha}{dt} \approx 0, \quad \frac{de_\beta}{dt} \approx 0 \quad (16)$$

From (8), (9) and (16), a PMSM can be modeled as in (17). The space-state model in (17) considers the $\alpha\beta$ stator currents and back-electromotive forces as the state variables, while the stator currents are the system outputs.

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} &= \begin{bmatrix} -r/l & 0 & -1/l & 0 \\ 0 & -r/l & 0 & -1/l \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} \\ &+ \begin{bmatrix} 1/l & 0 & 0 & 0 \\ 0 & 1/l & 0 & 0 \end{bmatrix}^T \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \\ \mathbf{y} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} \end{aligned} \quad (17)$$

The Luenberger observer can be applied to estimate the state vector $[i_\alpha \ i_\beta \ e_\alpha \ e_\beta]^T$ (and the back-electromotive forces) in (17). If e_α and e_β are known, equation (18) can be used to get the electrical angle:

$$\begin{aligned} \frac{e_\beta}{e_\alpha} &= \frac{\sin(\theta_e)}{\cos(\theta_e)} \\ &= \tan(\theta_e) \\ &= \tan(n\theta) \end{aligned} \quad (18)$$

However, the assumption in (16) is only valid for low speeds, due to the back-electromotive forces depend on the mechanical speed ω_m . In consequence, the estimation of the back electromotive forces is inaccurate for high speeds.

3 Proposed Speed Estimator

3.1 Improved Luenberger Observer

This paper proposes an improvement in the estimation of the back-electromotive forces, by redefining the system to be observed. The derivatives of e_α and e_β are:

$$\begin{aligned} \frac{de_\alpha}{dt} &= \frac{d[n\phi\omega_m \cos(n\theta)]}{dt} \\ &= n\phi \left[\frac{d\omega_m}{dt} \cos(n\theta) - \omega_m (n\omega_m) \sin(n\theta) \right] \\ &= n\phi \frac{d\omega_m}{dt} \cos(n\theta) - n^2\phi\omega_m^2 \sin(n\theta) \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{de_\beta}{dt} &= \frac{d[n\phi\omega_m \sin(n\theta)]}{dt} \\ &= n\phi \left[\frac{d\omega_m}{dt} \sin(n\theta) + \omega_m (n\omega_m) \cos(n\theta) \right] \\ &= n\phi \frac{d\omega_m}{dt} \sin(n\theta) + n^2\phi\omega_m^2 \cos(n\theta) \end{aligned} \quad (20)$$

Let assume that the acceleration is small during the operation of the PMSM (i.e., $d\omega_m/dt \approx 0$). Therefore, (19) and (20) can be approximated as follows:

$$\begin{aligned} \frac{de_\alpha}{dt} &\approx -n^2\phi\omega_m^2 \sin(n\theta) \\ &\approx -n\omega_m [n\phi\omega_m \sin(n\theta)] \\ &\approx -n\omega_m e_\beta \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{de_\beta}{dt} &\approx -n^2\phi\omega_m^2 \cos(n\theta) \\ &\approx -n\omega_m [n\phi\omega_m \cos(n\theta)] \\ &\approx -n\omega_m e_\alpha \end{aligned} \quad (22)$$

Combining (8), (9), (21) and (22), it is possible to get a better space-state model than (17) in order to estimate the back-electromotive forces:

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} &= \begin{bmatrix} -r/l & 0 & -1/l & 0 \\ 0 & -r/l & 0 & -1/l \\ 0 & 0 & 0 & -n\omega_m \\ 0 & 0 & n\omega_m & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} \\ &+ \begin{bmatrix} 1/l & 0 & 0 & 0 \\ 0 & 1/l & 0 & 0 \end{bmatrix}^T \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \\ \mathbf{y} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} \end{aligned} \quad (23)$$

Observe that (23) is a nonlinear system in due to the presence of the terms $\pm n\omega_m$ in the state matrix. In order to solve this problem, it is possible to linearize (23) respect to an average speed, replacing ω_m by ω_{av} . Hence, the following space-state model is obtained:

$$\frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} = \begin{bmatrix} -r/l & 0 & -1/l & 0 \\ 0 & -r/l & 0 & -1/l \\ 0 & 0 & 0 & -n\omega_{av} \\ 0 & 0 & n\omega_{av} & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} + \begin{bmatrix} 1/l & 0 & 0 & 0 \\ 0 & 1/l & 0 & 0 \end{bmatrix}^T \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ e_\alpha \\ e_\beta \end{bmatrix} \quad (24)$$

The average speed (ω_{av}) is defined as the average between the maximum and minimum steady-state mechanical speed expected during the operation of the motor. The space-state model in (24) is used in the improved Luenberger observer.

3.2 Project of the Feedback Matrix through LMI

The dynamics of the proposed modified Luenberger observer also depends on (15), but using the state matrix and output matrix in (24), which can be defined as $\mathbf{A}_o$ and $\mathbf{C}_o$, respectively. According to Lyapunov theory, the system in (15) is asymptotically stable if exists a positive definite matrix $\mathbf{Q}$ ($\mathbf{Q} > \mathbf{0}$) such as:

$$(\mathbf{A}_o - \mathbf{L}\mathbf{C}_o)\mathbf{Q} + \mathbf{Q}(\mathbf{A}_o - \mathbf{L}\mathbf{C}_o)^T < \mathbf{0} \quad (25)$$

The LMI in (25) can be solved making $\mathbf{P} = \mathbf{Q}^{-1}$ and $\mathbf{G} = \mathbf{P}\mathbf{L}$. Thus, the LMI in (25) can be rewritten as follows (Assunção, 2009; Gahinet, 1995):

$$\mathbf{P}\mathbf{A}_o + \mathbf{A}_o^T\mathbf{P} - (\mathbf{G}\mathbf{C}_o + \mathbf{C}_o^T\mathbf{G}^T) < \mathbf{0} \quad (26)$$

Besides, the closed-loop poles of the proposed estimator can be set within the region in Fig. 1, defined as $D(v, \rho, \phi)$, through the following LMIs:

$$\mathbf{P}\mathbf{A}_o + \mathbf{A}_o^T\mathbf{P} - (\mathbf{G}\mathbf{C}_o + \mathbf{C}_o^T\mathbf{G}^T) + 2v\mathbf{P} < \mathbf{0} \quad (27)$$

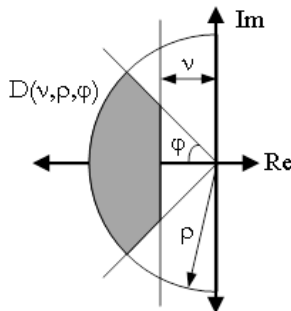


Figure 1. $D(v, \rho, \phi)$.

$$\begin{bmatrix} -\rho\mathbf{P} & \mathbf{P}\mathbf{A}_o - \mathbf{G}\mathbf{C}_o \\ \mathbf{A}_o^T\mathbf{P} - \mathbf{C}_o^T\mathbf{G}^T & -\rho\mathbf{P} \end{bmatrix} < \mathbf{0} \quad (28)$$

$$\begin{bmatrix} \sin(\phi)(\mathbf{P}\mathbf{A}_o + \mathbf{A}_o^T\mathbf{P} - \mathbf{G}\mathbf{C}_o - \mathbf{C}_o^T\mathbf{G}^T) \\ \cos(\phi)(\mathbf{A}_o^T\mathbf{P} - \mathbf{P}\mathbf{A}_o + \mathbf{G}\mathbf{C}_o - \mathbf{C}_o^T\mathbf{G}^T) \\ \cos(\phi)(\mathbf{P}\mathbf{A}_o - \mathbf{A}_o^T\mathbf{P} - \mathbf{G}\mathbf{C}_o + \mathbf{C}_o^T\mathbf{G}^T) \\ \sin(\phi)(\mathbf{P}\mathbf{A}_o + \mathbf{A}_o^T\mathbf{P} - \mathbf{G}\mathbf{C}_o - \mathbf{C}_o^T\mathbf{G}^T) \end{bmatrix} \quad (29)$$

4 Results

The proposed estimator was simulated in MATLAB/SIMULINK. The parameters of the PMSM and the observer are listed in Tables 1 and 2.

Table 1. Parameters of the PMSM.

Parameter	Value
Stator resistor	0.85 Ω
DQ-stator inductance	6 mH
Equivalent flux of the magnets	0.148 Wb
Rotor inertia	5x10 ⁻³ kg.m ²
Coefficient of friction	5x10 ⁻⁴ N.m.s
Number of pairs of poles	3

Table 2. Parameters of the Proposed Observer.

Parameter	Value
v	4000
ρ	7000
ϕ	1 rad
ω_{av}	70 rad/s

With these parameters, the observation matrix $\mathbf{L}$ of the proposed controller has the following values:

$$\mathbf{L} = \begin{bmatrix} 9251.90 & -93.42 \\ 93.42 & 9251.90 \\ -1.57 \times 10^5 & 6625.90 \\ -6625.90 & -1.57 \times 10^5 \end{bmatrix} \quad (30)$$

The performance of the proposed controller is compared with a conventional Luenberger observer (considering the derivatives of e_α and e_β equal to zero), using the space-state model in (17). The feedback matrix of the conventional Luenberger observer, denoted by $\mathbf{L}_{conv}$, was found using the same parameters and methodology that were used to calculate $\mathbf{L}$:

$$\mathbf{L}_{conv} = \begin{bmatrix} 9251.90 & 0 \\ 0 & 9251.90 \\ -1.57 \times 10^5 & 0 \\ 0 & -1.57 \times 10^5 \end{bmatrix} \quad (31)$$

Figs. 2, 3 and 4 shows the simulations result for $\sin(\theta_e)$ for speeds of 30 rad/s, 70 rad/s and 125 rad/s, respectively. In all these cases, the proposed Luenberger estimator gives more accurate estimations than the conventional Luenberger observer. Luenberger estimators have some problems for low mechanical speed. However, Fig. 2 shows that the estimation using the proposed algorithm is more accurate.

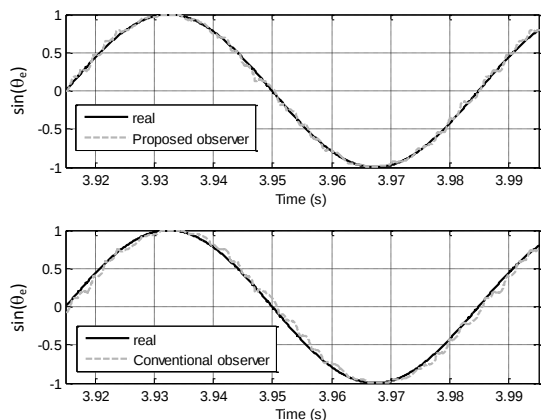


Figure 2. Estimation of $\sin(\theta_e)$ for $\omega_m = 30$ rad/s.

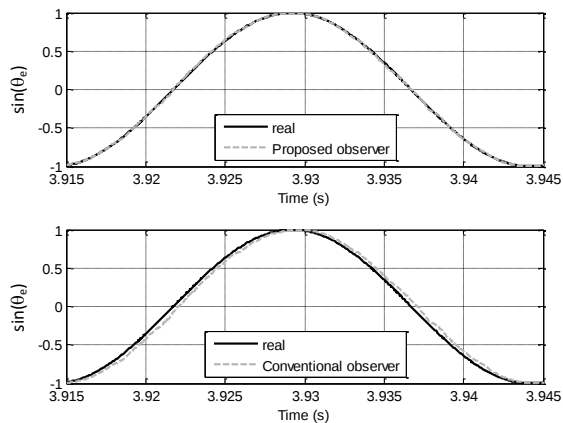


Figure 3. Estimation of $\sin(\theta_e)$ for $\omega_m = 70$ rad/s.

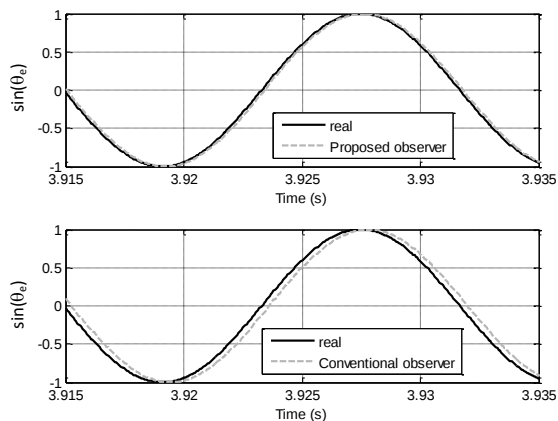


Figure 4. Estimation of $\sin(\theta_e)$ for $\omega_m = 125$ rad/s.

Figs. 5, 6 and 7 show the simulation results for the same mechanical speeds and the same observers used in the first three simulation tests, but considering an

attenuation of 10% in the electrical parameters of the PMSM. These results indicate that the proposed estimator is robust against parameter variation.

Fig. 7 shows the results for a constant acceleration of 200 rad/s² and considering a variation of 10% in the electrical parameters. Even in those conditions, the proposed Luenberger observer gives an accurate estimation.

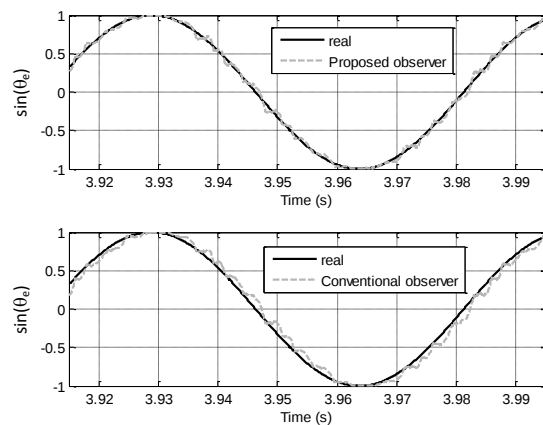


Figure 4. Estimation of $\sin(\theta_e)$ for $\omega_m = 30$ rad/s and parameter variation.

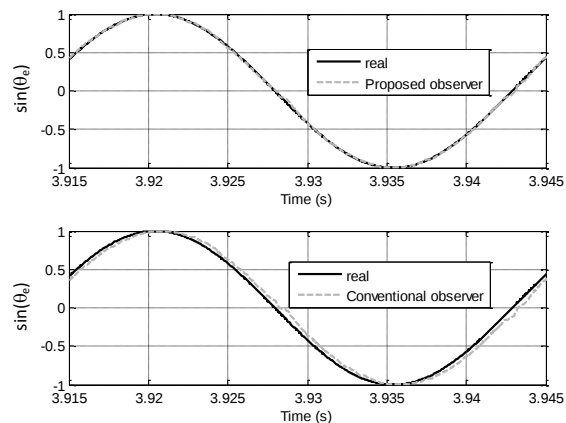


Figure 5. Estimation of $\sin(\theta_e)$ for $\omega_m = 70$ rad/s and parameter variation.

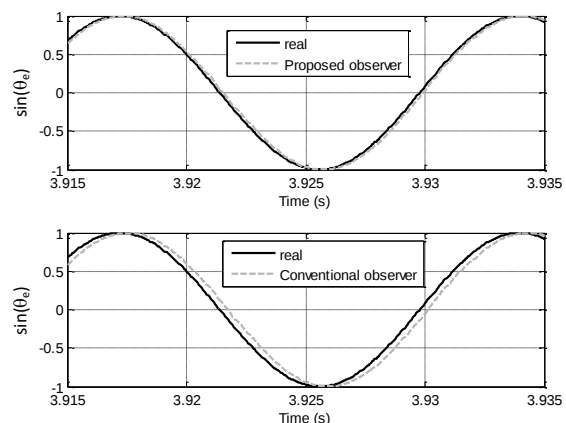


Figure 6. Estimation of $\sin(\theta_e)$ for $\omega_m = 125$ rad/s and parameter variation.

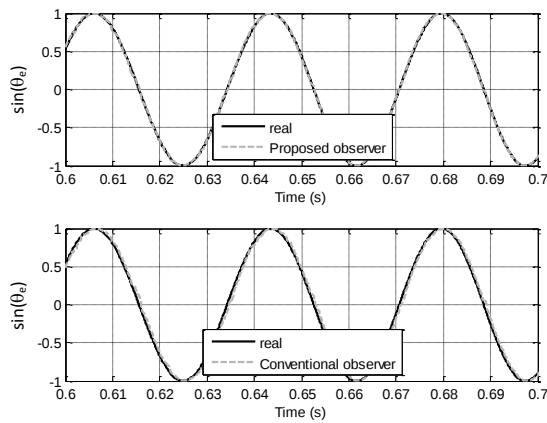


Figure 7. Estimation of $\sin(\theta_e)$ for an acceleration of 200 rad/s^2 and parameter variation.

4 Conclusion

This paper describes a modification of the Luenberger observer in order to improve the accuracy of the angle estimation for a PMSM. Simulation results show that the proposed estimator gives more accurate estimations on the back electromotive forces, and it is robust against parameter variation. As a result, the angular position and speed can be better estimated. As future work, the proposed Luenberger estimator will be implemented experimentally.

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