APPLICATION OF ROBUST SWITCHED CONTROLLERS IN ACTIVE SUSPENSION SYSTEM

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Abstract— In this paper, some results concerned with the control of uncertain switched linear systems using Lyapunov-Metzler inequalities and properties of Strictly Positive Real (SPR) systems are presented. A practical implementation is realized with the design of a robust controller, proposed in this paper, based on switched static state feedback and using Linear Matrix Inequalities (LMIs), in an active suspension system manufactured by the Quanser®. For the implementation, it is supposed that the active suspension system is subject to uncertainties in the model. Performance indices such as decay rate and restriction in the norm of controller gains are considered in the design. The results show an adequate performance and an efficient solution for this control problem.

Keywords— Robust Switched Control, LMIs, Uncertain Switched System, Strictly Positive Real Systems, Lyapunov-Metzler Inequalities.

Resumo— Neste artigo, são apresentados alguns resultados relacionados ao controle de sistemas lineares chaveados incertos usando as desigualdades de Lyapunov-Metzler e as propriedades de sistemas Estritamente Reais Positivos (SPR). Uma implementação prática é realizada com o projeto de um controlador robusto, proposto neste trabalho, baseado em realimentação com comutação estática do estado e usando Desigualdades Matriciais Lineares (LMIs), em um sistema de suspensão ativa, fabricado pela Quanser®. Para a aplicação, supõe-se que o sistema de suspensão ativa está sujeito a incertezas no modelo. Os índices de desempenho, tais como taxa de decaimento e restrição na norma dos ganhos do controlador são considerados no projeto. Os resultados mostram um desempenho adequado e uma solução eficiente para este problema de controle.

Palavras-chave— Controle Chaveado Robusto, LMIs, Sistema Chaveado Incerto, Sistemas Estritamente Reais Positivos, Desigualdades de Lyapunov-Metzler.

1 Introduction

Switched systems are dynamical systems composed of several subsystems switching among themselves according to a decision rule. This kind of dynamical systems exist in a variety of engineering applications. An important stability problem concerning switched systems, that has been studied, is the problem of finding conditions for the existence of a stabilizing switching signal for a given switched system (DeCarlo et al., 2000). The main idea of this paper is the design of Variable Structure Controller (VSC) based on the Lyapunov-Metzler (LM) inequalities proposed by Geromel and Colaneri (Geromel and Colaneri, 2006) and the stability properties of the Strictly Positive Real (SPR) systems. It is known that a linear time-invariant system is SPR if it satisfies the following properties. It is a passive system, is asymptotically stable, and all its transmission zeros have negative real parts. The SPR synthesis has been studied for various purposes, in the specialized literature is possible to find works that report its use in Linear Time-Invariant (LTI) systems such as the design of VSC systems (Teixeira et al., 2002; Hung et al., 1993), and in the output feedback tracking of uncertain systems (Hsu et al., 2001).

A sufficient condition to apply the SPR synthesis is that, given a LTI plant $\{A, B, C\}$,which is controllable and observable, find constant matrices F and K_0 such that the controlled system $\{A - BK_0C, B, FC\}$ is SPR. For plants with the number of inputs m equal to the number of outputs p, this problem has solution if and only if all transmission zeros of the plant have negative real parts and $det(CB) \neq 0$ (Owens et al., 1987). The procedure to solve this problem using Linear Matrix Inequalities (LMIs) can be easily solved with available software. Moreover, using this tool, we can consider other design specifications like plant uncertainties, decay rate (related to the setting time), and output and input constraints (Teixeira et al., 2002; Bernussou et al., 1999).

In (Geromel and Colaneri, 2006), a stability analysis and control design is proposed for the continuous-time switched linear system given by $\dot{x} = A_{\sigma(t)}x(t), \quad x(0) = x_0$, where $x(t) \in \mathbb{R}^n$ is the state vector, $\sigma(t)$ is the switching control rule, and x_0 is the initial condition. The design of this control law was described as the solution of LM inequalities. This method was applied in the design of a switching control law of a comfort-oriented semiactive suspension in road vehicles and a better performance was obtained when compared with other available methods (Geromel et al., 2008). In (Deaecto et al., 2010), it is studied a novel procedure for switched affine systems control design especially developed to deal with switched converters where the main goal is to attain a set of equilibrium points. More recent researches in this context can be found in (Duan and Wu, 2012; Contzen and Raisch, 2014; Hsu et al., 2015).

This work aims mainly to confirm the good performance of the proposed methodology in (Cardim et al., 2016). Thus, a practical implementation was realized in an active suspension system manufactured by the Quanser®, for that, failures and uncertainties in the plant model were considered. The computer simulations were carried out by using the software package *YALMIP* (Löfberg, 2004), with the solver *LMIlab* (Gahinet et al., 1995).

For convenience, some notations that will be used at work shall be established: $\mathbb{K}_N = \{1, 2, ..., N\}$, $N \in \mathbb{N}$, $A_{\sigma(t)}(\alpha) = A_\sigma(\alpha)$, $K(\alpha) = \sum_{i=1}^r \alpha_i K_i$, for $\alpha_i \geqslant 0$ and $\sum_{i=1}^r \alpha_i = 1$.

2 Uncertain Switched Linear System

As some parameters of the plant are not completely known, let us consider the following uncertain switched linear system:

$$\dot{x}(t) = A_{\sigma(t)}(\alpha)x(t) + B_{\sigma(t)}(\alpha)u_a(t), \tag{1}$$

$$y(t) = C_{\sigma(t)}x(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\sigma(t)$ is the switching control rule, $x(0) = x_0$ is the initial condition, $u_a(t)$ is an additional control input and $y(t) \in \mathbb{R}^p$ is the output. The matrices $A(\alpha) \in \mathbb{R}^{n\times n}$ and $B(\alpha) \in \mathbb{R}^{n\times m}$ are given by

$$A_{\sigma(t)}(\alpha) = A_\sigma(\alpha) = \sum_{k=1}^r \alpha_k A_{\sigma k}, \tag{3}$$

$$B_{\sigma(t)}(\alpha) = B_\sigma(\alpha) = \sum_{k=1}^r \alpha_k B_{\sigma k}, \tag{4}$$

where $A_{\sigma k}$ and $B_{\sigma k}$ are known matrices

$$\begin{array}{l} A_{\sigma k} \in \{A_{1k}, A_{2k}, ..., A_{Nk}\}, \\ B_{\sigma k} \in \{B_{1k}, B_{2k}, ..., B_{Nk}\}, \end{array} \tag{5}$$

$$\alpha \in [\alpha_1, \alpha_2, ..., \alpha_r]^T, \sum_{k=1}^r \alpha_k = 1, \alpha_k \geq 0, k \in \mathbb{K}_r. \tag{6}$$

In the matrices $A_{\sigma(t)}(\alpha), B_{\sigma(t)}(\alpha)$, the vector α is associated with the polytopic uncertainties (or structural failures) of the plant and $\sigma(t)$ is the switching strategy, as described earlier.

2.1 LM-SPR Synthesis for Uncertain Switched Linear Plants

Based in the control synthesis for switched linear systems through the so-called LM-SPR in (Cardim et al., 2008), let us consider the Problem 1 to describe the proposed synthesis for uncertain switched linear plants in (Cardim et al., 2016).

Problem 1 *Given the uncertain switched linear system (1)–(6), with $B_\sigma(\alpha)$ and $C_\sigma(\alpha)$ as defined in (4), where $rank(B_\sigma(\alpha)) = m$ for all α defined in (6), then find necessary and sufficient conditions for the existence of matrices $F_\sigma(\alpha) \in \mathbb{R}^{m\times p}$ and $\bar{K}_\sigma(\alpha) \in \mathbb{R}^{m\times n}$, $\bar{K}_\sigma(\alpha) = \sum_{k=1}^r \alpha_k \bar{K}_{\sigma k}$ and $F_\sigma(\alpha) = \sum_{k=1}^r \alpha_k F_{\sigma k}$, and $\bar{K}_{ik}$, F_{ik} with ($i \in \mathbb{K}_N$, $k \in \mathbb{K}_r$) are constant matrices that make the feedback system shown in Figure 1, with input $\tilde{u}(t)$ and output $\tilde{y}(t)$, an LM-SPR system.*

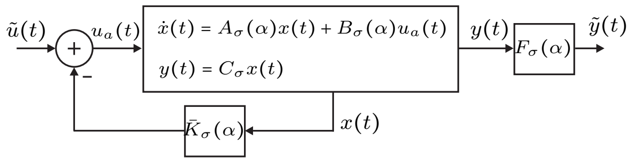

Figure 1: Feedback uncertain linear system for LM-SPR synthesis described in Problem 1.

A solution for Problem 1 is proposed in the next theorem which is an extension of the results presented for linear time invariant and known systems in (Cardim et al., 2008; Cardim et al., 2011), now for uncertain switched systems. Furthermore let us consider an important performance index, called decay rate. In (Boyd et al., 1994), the decay rate is defined as the largest real constant γ, $\gamma > 0$ such that

$$\lim_{t\to\infty} e^{\gamma t} \|x(t)\| = 0. \tag{7}$$

Lemma 1 *(Boyd et al., 1994) the controlled system (1)–(2) has a decay rate at least equal to $\gamma, \gamma > 0$ if for all trajectory of $x(t)$, $t \geqslant 0$ there exist $P_i = P_i^T > 0, i \in \mathbb{K}_N$ such that $v(x) = \min_{i=1,2...,N} x^T P_i x$ satisfies the condition*

$$\dot{V}(x) \leq -2\gamma V(x). \tag{8}$$

Proof: see (Boyd et al., 1994) for details. □

Theorem 1 *Assume that $p = n$ and $det(C_i) \neq 0$, for $i \in \mathbb{K}_N$. Then, Problem 1 can be solved, with decay rate greater than or equal to γ, if and only if there exist matrices $X_i = X_i^T, M_{iq}$ and scalars $\pi_{ji} \geq 0$ for $i \neq j$ and $\sum_{j=1}^N \pi_{ji} = 0$, $i, j \in \mathbb{K}_N$ $q \in \mathbb{K}_r$, such that (9) holds:*

$$\begin{bmatrix} -R_{ik} & X_i & & & \cdots & & X_i \\ X_i & \frac{X_1}{\pi_{1i}} & 0 & & \cdots & & 0 \\ & 0 & \ddots & & & & \\ & & & \frac{X_{i-1}}{\pi_{i-1,i}} & 0 & & \vdots \\ \vdots & \vdots & & 0 & \frac{X_{i+1}}{\pi_{i+1,i}} & & \\ & & & & & \ddots & 0 \\ X_i & 0 & & \cdots & & 0 & \frac{X_N}{\pi_{Ni}} \end{bmatrix} > 0,$$

$$R_{ik} = A_{ik}X_i + X_i A_{ik}^T - B_{ik}M_{iq} - M_{iq}^T B_{ik}^T + \pi_{ii}X_i + 2\gamma X_i, \tag{9}$$

for all $i \in \mathbb{K}_N$ and $k, q \in \mathbb{K}_r$. Furthermore, if (9) are feasible, with a solution given by $X_i = P_i^{-1}$, M_{ik}

and π_{ji}, the following Matrices F_{ik} and K_{ik} are a solution of Problem 1:

$$F_{ik} = B_{ik}^T P_i C_i^{-1}, \tag{10}$$

$$K_{ik} = M_{ik} P_i C_i^{-1}, \quad i \in \mathbb{K}_N, \quad k \in \mathbb{K}_r. \tag{11}$$

Proof: See (Cardim et al., 2016) for details. □

The Theorem 1 solves Problem 1 and is a generalization of the method proposed in (Cardim et al., 2011) for known linear time-invariant plants. Thus, this result can be useful in the design of switched static state feedback controllers for uncertain switched plants, based on SPR synthesis. Another performance index that can be added in the LM-SPR synthesis method, that allows the specification of an upper bound for the norm of the controller gains, as described later.

Lemma 2 *(Šiljak and Stipanovic, 2000) The norm of the gains, such that $K_{ik}K_{ik}^T \le \eta\eta_x^2$ is guaranteed if there are constants $\eta > 0$ and $\eta_x > 0$ such that the LMIs of Theorem 1, with (12), hold.*

$$\begin{bmatrix} \eta_x I_n & I_n \\ I_n & X_i \end{bmatrix} \geqslant 0, \ \begin{bmatrix} \eta I_n & M_{ik}^T \\ M_{ik} & I_m \end{bmatrix} \geqslant 0, \ i \in \mathbb{K}_N, k \in \mathbb{K}_r. \tag{12}$$

Proof: See (Šiljak and Stipanovic, 2000). □

2.2 Application using state feedback

A design method of switched controllers for the switched system (1)–(6) is presented. To find the feedback gains, the control law uses two stages. The first stage is based on the procedures cited before and chooses an index

$$\sigma = arg \min_{i=1,2\ldots,N} x^T P_i x, \tag{13}$$

where $P_i = P_i^T > 0$ and the Lyapunov function is equal to $v(x) = x^T P_\sigma x$. The idea of the second stage is the minimization of the time derivative of the Lyapunov function, through the selection of the controller gain, which belongs to the set of gains $\{K_{\sigma k}, k \in \mathbb{K}_r\}$ where σ was obtained in the first stage. This stage uses auxiliary symmetric matrices $Q_{ik}, i \in \mathbb{K}_N, k \in \mathbb{K}_r$ and selects an index

$$\nu = arg \min_{q=1,2..,r} x^T Q_{\sigma q} x, \tag{14}$$

Therefore, considering the indices σ (13) and ν (14), the switched controller is defined as follows:

$$u_a(t) = u_{\sigma\nu}(t) = -K_{\sigma\nu} x(t). \tag{15}$$

From (1)-(6), the controlled system is given by

$$\begin{aligned} \dot{x} &= A_\sigma(\alpha)x(t) + B_\sigma(\alpha)u_{\sigma\nu}(t), \\ x(0) &= x_0. \end{aligned} \tag{16}$$

The proposed method in this paper, presented in the next theorem, was based on (Cardim et al., 2016). Certainly, the proposed theorem was adapted to satisfy more LMIs than the Theorem 5 presented in (Cardim et al., 2016) to avoid unstable modes showed in the practical implementation owing to the delay of the control action of the real time system.

Theorem 2 *Consider the system described by (16). If there exist $X_i = X_i^T > 0 \in \mathbb{R}^{n\times n}$, symmetric matrices $\bar{Z}_{ik} \in \mathbb{R}^{n\times n}$ and $\bar{Q}_{iq} \in \mathbb{R}^{n\times n}$, matrices $M_{jq} \in \mathbb{R}^{m\times n}$ and scalars $\pi_{ji} > 0$ for $i \ne j$ and $\sum_{j=1}^N \pi_{ji} = 0$, such that*

$$-B_{ik}M_{jq} - M_{jq}^T B_{ik}^T - \bar{Z}_{ik} - \bar{Q}_{iq} \le 0, \tag{17}$$

$$\begin{aligned} A_{ik}X_i + X_i A_{ik}^T + \bar{Z}_{ik} + \bar{Q}_{ik} + \sum_{l=1}^N \pi_{li} X_i P_l X_i + \\ 2\gamma X_i < 0, \end{aligned} \tag{18}$$

hold for all $i, j, l \in \mathbb{K}_N$, k, $q \in \mathbb{K}_r$. Then the controller (15) with the switched control law (13) and (14), $Q_{iq} = X_i^{-1}\bar{Q}_{iq}X_i^{-1}$, $K_{ij} = M_{ij}P_iC_i^{-1}$ and $P_i = X_i^{-1}$ for $i \in \mathbb{K}_N$ and $q \in \mathbb{K}_r$ makes the equilibrium point $x(t) = 0$ of the system (16) globally asymptotically stable. Furthermore, the decay rate is equal to or greater than γ.

Proof: Assume that there exist matrices $Z_{ik} = Z_{ik}^T \in \mathbb{R}^{n\times n}$ and $Q_{iq} = Q_{iq}^T \in \mathbb{R}^{n\times n}$, such that

$$-P_iB_{ik}K_{jq}C_i - C_i^T K_{jq}^T B_{ik}^T P_i \le Z_{ik} + Q_{iq}. \tag{19}$$

The following conditions are sufficient for the solution of Problem 1, with decay rate specification:

$$\begin{aligned} P_i(A_{ik} - B_{ik}K_{jq}C_i) + (A_{ik} - B_{ik}K_{jq}C_i)^T P_i + \\ \sum_{l=1}^N \pi_{li}P_i + 2\gamma P_i < 0, \end{aligned} \tag{20}$$

$$P_i = P_i^T > 0, \tag{21}$$

$$B_{ik}^T P_i = F_{ik}C_i, \tag{22}$$

for $i, j, l \in \mathbb{K}_N$, $k, q \in \mathbb{K}_r$, where $\pi_{ji} > 0$ for $i \ne j$ and $\sum_{j=1}^N \pi_{ji} = 0$. Therefore, it follows that:

$$\begin{aligned} D^+(v(x)) &\leqslant \sum_{k=1}^N \alpha_k x^T (P_iA_{ik} + A_{ik}^T P_i - P_iB_{ik}K_{jq}C_i \\ &\quad -C_i^T K_{jq}^T B_{ik}^T P_i)x + x^T \sum_{l=1}^N \pi_{li}P_l x + x^T 2\gamma P_i x \\ &\leqslant \sum_{k=1}^N \alpha_k x^T (P_iA_{ik} + A_{ik}^T P_i + Z_{ik} + Q_{iq})x + \\ &\quad x^T \sum_{l=1}^N \pi_{li}P_l x + x^T 2\gamma P_i x. \end{aligned} \tag{23}$$

Considering that $x^T Q_{\sigma\nu} x \le \sum_{k=1}^N \alpha_k x^T Q_{\sigma k} x$

$$\begin{aligned} D^+(v(x)) &\leqslant \sum_{k=1}^N \alpha_k x^T (P_iA_{ik} + A_{ik}^T P_i + Z_{ik})x + \\ &\quad x^T \sum_{l=1}^N \pi_{li}P_l x + \min_{q=1,..,r} x^T Q_{\sigma q} x + x^T 2\gamma P_i x \\ &\leqslant \sum_{k=1}^N \alpha_k x^T (P_iA_{ik} + A_{ik}^T P_i + Z_{ik} + Q_{ik})x + \\ &\quad x^T \sum_{l=1}^N \pi_{li}P_l x + x^T 2\gamma P_i x < 0. \end{aligned} \tag{24}$$

Therefore, from (24), $D^{+}(v(x)) < 0$ holds if the following condition holds:

$$P_i A_{ik} + A_{ik}^T P_i + Z_{ik} + Q_{ik} + \sum_{l=1}^{N} \pi_{li} P_l + 2\gamma P_i < 0, \quad (25)$$

for all $i \in \mathbb{K}_N$, $k, q \in \mathbb{K}_r$. Premultiplying and postmultiplying (19) and (25) by $X_i = P_i^{-1}$ and defining $\bar{Q}_{ik} = X_i Q_{ik} X_i$ and $\bar{Z}_{ik} = X_i Z_{ik} X_i$ one obtains (17) and (18). From (24) and (25) $x = 0$ is a globally asymptotically stable equilibrium point of the controlled system. From Lemma 1 and from (25) the decay rate is equal to or greater than γ. The proof is concluded. □

Note that, applying the Schur Complement in (18), and following the procedure used in equations (37)-(39) in (Cardim et al., 2016), one has:

$$\begin{bmatrix} -G_{ik} & X_i & & & \cdots & X_i \\ X_i & \frac{X_1}{\pi_{1i}} & 0 & & \cdots & 0 \\ & 0 & \ddots & & & \\ & & \frac{X_{i-1}}{\pi_{i-1,i}} & 0 & & \vdots \\ \vdots & \vdots & 0 & \frac{X_{i+1}}{\pi_{i+1,i}} & & \\ & & & & \ddots & 0 \\ X_i & 0 & \cdots & & 0 & \frac{X_N}{\pi_{Ni}} \end{bmatrix} > 0 \quad (26)$$

$G_{ik} = A_{ik} X_i + X_i A_{ik}^T + \bar{Z}_{ik} + \bar{Q}_{ik} + \pi_{ii} X_i + 2\gamma X_i$.

This procedure is used in the simulation of an Active Suspension System with the switching strategy given in (13)-(14) and the conditions given in Theorem 2.

3 Active Suspension System

Consider the active suspension system of a vehicle, with the schematic model given in Figure 2. This system is manufactured by Quanser® (Figure 3) (Quanser, 2009). The system consists of a set of two masses, calls of $M_s(\alpha)$ and M_{us}. The M_s mass represents $\frac{1}{4}$ of the total vehicle body and is supported by the spring k_s and by the damper b_s. The M_{us} mass corresponds the mass of the tire set and is supported by the spring k_{us} and by the damper b_{us}. The vibrations caused by irregularities in the street can be attenuated by the vehicle's active suspension system, represented by a motor (actuator) connected between the masses M_s and M_{us}, and controlled by the force F_c.

The dynamic model can be represented by the uncertain linear system given in (1), considering

$$x(t) = [(z_s - z_{us}) \quad \dot{z}_s \quad (z_{us} - z_r) \quad \dot{z}_{us}]^T,$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-\mathrm{k}_s}{M_s(\alpha)} & \frac{-b_s}{M_s(\alpha)} & 0 & \frac{b_s}{M_s(\alpha)} \\ 0 & 0 & 0 & 1 \\ \frac{\mathrm{k}_s}{M_{us}} & \frac{b_s}{M_{us}} & \frac{-\mathrm{k}_{us}}{M_{us}} & \frac{-b_s - b_{us}}{M_{us}} \end{bmatrix} x$$
$$+ \begin{bmatrix} 0 & \frac{\beta_{\sigma(t)}}{M_s(\alpha)} & 0 & \frac{-\beta_{\sigma(t)}}{M_{us}} \end{bmatrix}^T u. \quad (27)$$

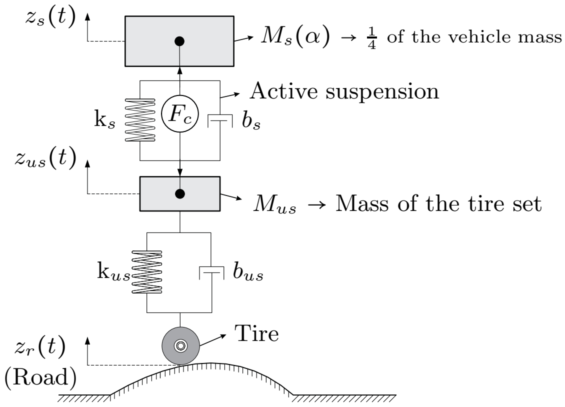

Figure 2: Schematic model.

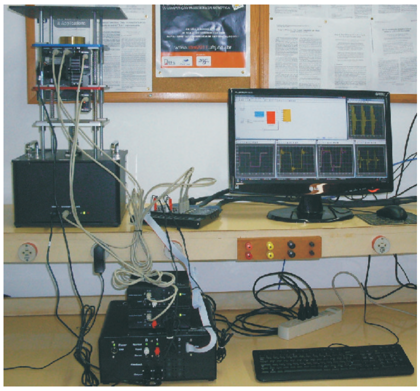

Figure 3: Active suspension system from Quanser®.

The respective values for the system parameters are shown in Table 1. In the physical model of the active suspension system there is a payload mass (brass) removable. It consists of two identical weight units, resulting in the mass $M_s(\alpha)$. Each unit has the weight of 0.4975kg, such that, the total mass corresponds to the 2.45kg, reported in the Table 1. Therefore, the M_s mass may assume values between 1.455kg (without the two weight units) and 2.45kg (with the two weight units). Thus, the M_s mass may be uncertain and belongs to the interval $1.455 \leq M_s \leq 2.45$(kg).

Table 1: Parameters of the system.

Symbol	Value
M_s	2,45 (Kg)
M_{us}	1 (Kg)
K_s	900 (N/m)
k_{us}	2500 (N/m)
b_s	7,5 (Ns/m)
b_{us}	5 (Ns/m)

4 Simulation Results

In the system simulation, it is considered M_s = 2.45kg with a limited uncertainty ±0.49kg, furthermore $\beta_1 = 0.5$ (when $\sigma(t) = 1$) and $\beta_2 = 1$ (when $\sigma(t) = 2$). For these parameters, the two matrices $A_1(\alpha)$ and $A_2(\alpha)$ are both stable.

Figure 4 shows a simulation of the feasible region, with constraints on the decay rate and norm of controller gains. To design the controller, these results may help the designer to find coefficients regions (π_{12} and π_{21}) for the controller design.

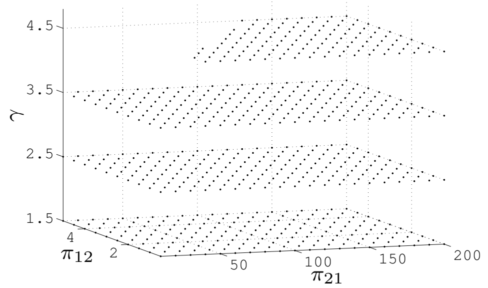

Figure 4: Feasible region, with $\eta\eta_x^2 = 16 \times 10^7$.

Considering the conditions given in Theorem 2, and considering the maximization of the $trace(X_1 + X_2)$, choosing $\pi_{21} = 160$, $\pi_{12} = 1.5$, and $\gamma = 3$, the method provides the following matrices:

$$P_1 = \begin{bmatrix} 184.9141 & 9.9532 & 185.0909 & 1.4469 \\ 9.9532 & 4.3807 & 0.3330 & 0.5887 \\ 185.0909 & 0.3330 & 226.1800 & 0.2152 \\ 1.4469 & 0.5887 & 0.2152 & 0.0842 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 191.0166 & 10.8667 & 191.2933 & 1.5912 \\ 10.8667 & 3.4326 & 4.0839 & 0.4713 \\ 191.2933 & 4.0839 & 224.3725 & 0.7115 \\ 1.5912 & 0.4713 & 0.7115 & 0.0697 \end{bmatrix},$$

$$\begin{aligned} K_{11} &= \begin{bmatrix} -636.49 & 71.68 & -878.05 & 9.17 \end{bmatrix}, \\ K_{12} &= \begin{bmatrix} -149.77 & 242.55 & -1141.94 & 31.12 \end{bmatrix}, \\ K_{21} &= \begin{bmatrix} -639.04 & 41.58 & -804.51 & 5.30 \end{bmatrix}, \\ K_{22} &= \begin{bmatrix} -150.40 & 169.46 & -936.01 & 22.02 \end{bmatrix}. \end{aligned}$$

Figures 5 and 6 show the simulations results considering as excitation signal (in the road profile $z_r(t)$) a square wave signal, with amplitude 0.04m, frequency of $\frac{1}{3}$Hz with pulse width of 50% in two cases: $M = 1.96$Kg ($0 \le t < 3$ s) and $M = 2.94$Kg ($t \ge 3$ s). Note that, as shown in Figure 5, the controlled system presented an adequate performance for the response of the position $z_s(t)$.

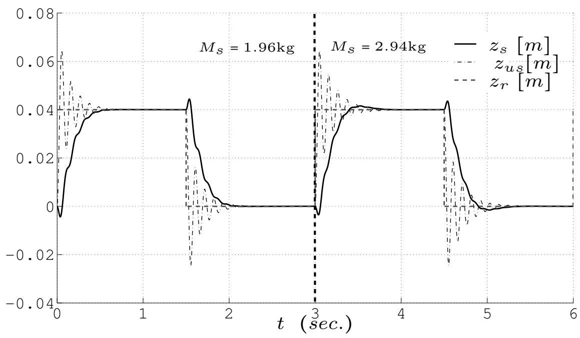

Figure 5: Simulation response with switching control strategy, $\eta\eta_x^2 = 16 \times 10^7$ and $\gamma = 3$.

5 Practical Implementation - Active Suspension Control

Practical results were obtained considering the same conditions of numerical simulation and the outcomes presented were obtained in Figures 7 and 8. In this implementation, it was considered the open-loop system in the range $0 \le t \le 6$ seconds and the controlled system in $6 < t \le 12$ seconds.

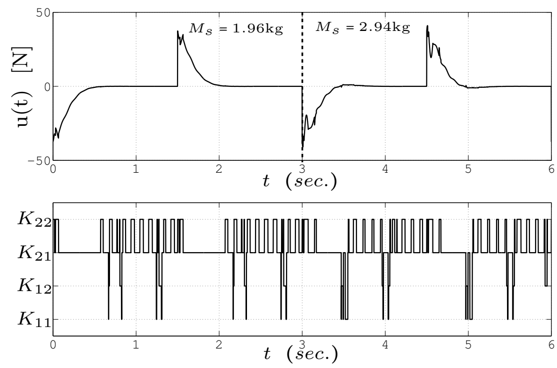

Figure 6: Control signal and gain selection $K_{\sigma\nu}(t)$ at each moment.

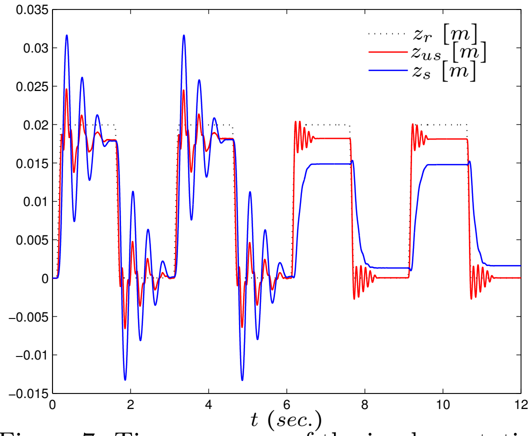

Figure 7: Time response of the implementation.

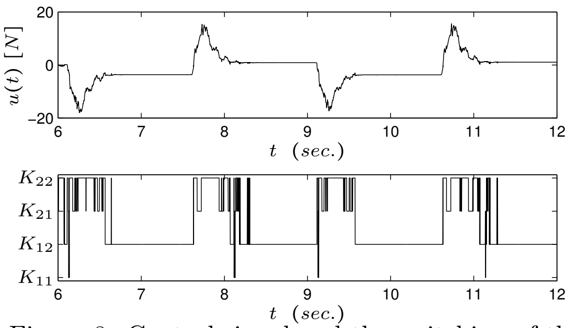

Figure 8: Control signal and the switching of the gain $K_{\sigma\nu}(t)$ in the practical implementation.

6 Conclusions

A control design method for continuous-time uncertain switched linear systems using a state feedback switching strategy was proposed in this paper, based on (Cardim et al., 2016). New conditions were obtained and the theory developed is illustrated through of simulation and implementation in the active suspension control system. The results obtained with the proposed method showed adequate performance.

It is important to mention that the presence of the offset, showed in Figure 7, in spite of the adequate behavior of the response signal, is due to the nonlinearities that occur in the real model of the active suspension system, such as friction and dead band. In this case, the possibility of improving the calibration and study the nonlinearities of the system can be considered. This subject is suggested for a future research.

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