HARVESTING WITH STOCHASTIC CONTROL: WHEN PARAMETERS ARE BADLY KNOWN

Vinícius Lima Silva*, João Bosco Ribeiro do Val*, Rafael Fontes Souto†

*Departamento de Sistemas e Energia
Faculdade de Engenharia Elétrica e de Computação
Universidade Estadual de Campinas
Campinas, SP, Brasil

†Departamento Acadêmico de Eletrotécnica
Universidade Tecnológica Federal do Paraná
Curitiba, PR, Brasil

Emails: vlima@dt.fee.unicamp.br, jbosco@fee.unicamp.br, rfsouto@utfpr.edu.br

Abstract— Harvesting management is often affected by uncertainties such as those regarding the assessment of stock levels and the growth dynamics of commercially exploited fisheries. Previous works have pointed this out and developed management techniques aimed at minimizing the effects uncertainties have on fisheries management. In this paper we propose to use the CVIU — Control Variation Increases the Uncertainties — approach, developed to control uncertain stochastic systems, to the harvesting problem. Numerical simulations show the proposed CVIU approach performs better than the LQG approach in scenarios with large uncertainties.

Keywords— Stochastic Control, Risk, Uncertainty, Harvesting, Fisheries.

Resumo— O gerenciamento de pescas normalmente está sujeito a incertezas tais como aquelas afetando o monitoramento do nível dos estoques e a dinâmica de crescimento das reservas pesqueiras exploradas comercialmente. Trabalhos anteriores salientaram isso e apresentarem técnicas com o objetivo de minimizar os efeitos das incertezas sobre o gerenciamento de recursos pesqueiros. Neste trabalho nós propomos o uso da abordagem VCAI — Variação do Controle Aumenta a Incerteza —, desenvolvida para controlar sistemas estocásticos incertos, no estudo do problema de pesca. Simulações numéricas mostram que a abordagem proposta oferece um desempenho melhor que o controlador LQG em cenários com incertezas grandes.

Palavras-chave— Controle Estocástico, Risco, Incertezas, Pesca.

1 Introduction

According to reports from the Fisheries Department of the Food and Agriculture Organization of the United Nations FAO (2014) and nature conservation groups such as WWF, the fraction of world fisheries harvested at an unsustainable rate has increased in the last decades. An all time high of 32.5% overfished commercial stocks was observed in 2008, followed by a slight decrease to 28.8% in 2011. In the same year about 61.3% of the stocks were fully fished while the underfished stocks accounted for 9.9%. Meanwhile the world per capita apparent fish consumption increased from an average of 9.9kg in the 1960s to 18.9kg in 2010.

These numbers highlight the importance of studying optimal fisheries management techniques which take the conservation of fish stocks into account, as studied in Clark and Kirkwood (1996), Roughgarden and Smith (1996) and Sethi et al. (2005) as a few samples. As a matter of fact, many governments around the world actively seek to maintain the fisheries harvest within a sustainable range, usually through the use of seasonal harvesting quotas. However, as previously pointed out in Roughgarden and Smith (1996), uncertainties regarding the modeling and monitoring of fisheries dynamics may make the use of such harvest quotas ineffective. As an example, Roughgarden cites the case of Newfoundland's cod fishery collapse [Walters and Maguire (1996)]. Even though the harvesting quotas set by Canada's fishery authorities were obeyed by Newfoundland's fishers, the fishery still collapsed — as of 2015, the cod stocks in the area have not yet rebounded. In Brazil, according to a study published by the Ministry of Environment [MMA (2006)], in the early 2000s 80% of the main fisheries were fully exploited, overfished or in recovery process. A loose inspection of the harvest of fisheries in the Brazilian coast and a lack of coordination between distinct government agencies responsible for the implementation of fishing policies[1] has aggravated the overfishing of Brazilian stocks.

As stressed in the works of Roughgarden and Smith (1996) and of Sethi et al. (2005), environmental variability, inadequate estimation of stock sizes and managerial uncertainty [Sethi et al. (2005)] are possible causes of the collapse of the canadian cod and other fisheries. These au-

[1]As reported in Azevedo and Pierri (2014), in 2011 the Ministry of Fisheries and Aquaculture issued new sardine fishing licences even though a previous decree by IBAMA, the Brazilian Institue of Environment and Renewable Resources, forbade the admittance of new sardine fishing boats. Within the 2010 mullet fishing season, despite IBAMA advocating the licensing of 60 fishing boats, the Ministry authorized 89 licences.

thors propose therefore harvest management techniques which aim to deal with such uncertainties. Sethi et al. (2005) design a model comprising the three sources of uncertainties mentioned before, whereas Roughgarden and Smith (1996) argue for a shift of the optimal harvesting point, as follows. Using the logistic function as a model for the growth of the fish biomass, $Z(t)$, we have $dZ(t) = (r\,(1 - Z(t)/K)\,Z(t) - h(t))\,dt$, where K is the environment carrying capacity. The maximum sustainable yield (MSY) then corresponds to $Z(t) = K/2$, however, this is an unstable equilibrium point and, as shown in Roughgarden and Smith (1996), unexpected shocks to the environment or demand may cause the fishery to collapse. Hence the paper proposes fisheries to be exploited at $Z(t) = 3K/4$, which provides lower immediate financial return but increases the stability of fish stocks.

We can therefore conclude that a sustainable fisheries management plan should include the analysis of uncertainties affecting the model chosen to portray the fish stock dynamics as well as the difficulties in implementing the desired actions. In this sense, the problem of fishery management with uncertain model is seen here as the problem of managing a renewable resource with poorly known growth dynamics, subject to an also badly known consumption rate. We assume the growth is (roughly) described by a logistic model with poorly known characteristic parameters. A control technique that fits in the scenario is the CVIU — acronym for Control Variations Increase the Uncertainties — approach, developed in Souto et al. (2013) and aimed at dealing with uncertain stochastic systems.

The paper is organized as follows. In section 2 we present an overview of growth models used in harvesting theory and, in section 3, a summary of the CVIU model. These concepts are applied to an optimal harvesting problem in 4, followed by numerical examples in 5 and a summary of the main findings in section 6.

2 Growth modelling

The growth of a single species population, as observed in nature, often presents an exponential trait, which has led to the modeling of growth of biological populations as an exponential process. The rate at which the population grows, however, is not constant [Murray (2002)]. Due to a limited availability of supplies in the environment, the population dynamics resembles a self-limiting process as the population size approaches the so called environmental carrying capacity and the growth rate slows down as the population size increases. Based on these observations, the logistic, Richards and Gompertz functions are often used to approximate the dynamic evolution of a single species population.

Denote the biomass amount at a time instant by $Z(t)$ and suppose the population at the same time is harvested at a rate $h(t)$ and the fishery manager estimation of the current stock size is affected by a random disturbance. We have, according to the logistic function,

$$dZ(t) = \left(r \left(1 - \frac{Z(t)}{K} \right) Z(t) - h(t) \right) dt + \sigma dW(t) \tag{1}$$

in which $K \geq 0$ is the carrying capacity of the environment, usually determined by the available resources [Murray (2002)], and $r \geq 0$, the slope of the curve at $Z = 0$, is the intrinsic growth rate. In a similar fashion, the Gompertz equation is given by

$$dZ(t) = \left(\alpha \log \left(\frac{K}{Z(t)} \right) Z(t) - h(t) \right) dt + \sigma dW(t) \tag{2}$$

Both Gompertz and logistic functions exhibit a dependence of the growth rate on the population size, as previously noted. The Gompertz equation, however, does not display the symmetry in growth seen in the logistic function.

3 Dealing with stochastic uncertainties: the CVIU approach

In du Pin et al. (2009), Souto et al. (2013), and Souto (2015), a strategy for controlling uncertain stochastic systems based on the idea that the Control Variation Increases the Uncertainties is analysed. According to these ideas, for a class of systems with highly uncertain nonlinear dynamics, it is reasonable to assume that, due to the limited knowledge about the system dynamics, a control action may lead the system to a region in the state space in which the uncertainty regarding the real state of the system may be even higher. Moreover, for some systems, specially in biology and economics, such as the one studied here or in economic dynamics [Brock et al. (2007)], it is not possible or reasonable to carry out experiments in order to attain a more accurate description of the system dynamics.

Within this framework the authors introduce the idea of a random disturbance term modulated by the intensity of the control variation [Souto et al. (2013)], which was later generalized to include disturbances modulated by deviations of the system state from the equilibrium point Souto and do Val (n.d.).

In this sense, we assume that the dynamics of the system we wish to control is described by a stochastic differential equation, as in Souto and do Val (n.d.)

$$dZ(t) = G(t, Z, v)dt + \sigma(t)dW(t) \tag{3}$$

for which $W = W(t)$ is a Brownian motion defined over the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, $G(t, Z, v)$ a nonlinear function and the control strategy $v = \{v(t)\}_{t\geq 0}$ a $\{\mathcal{F}_t\}$-adapted stochastic process taking its value in a compact set $\mathbb{U} \subset \mathbb{R}$. Moreover, supposing the system operates around an equilibrium point (Z_e, v_e), G is differentiable at (Z_e, v_e) and substituting for the variations $x(t) = Z(t) - Z_e$ and $u(t) = v(t) - v_e$, we can approximate the system dynamics by a local linear model

$$dZ(t) \simeq dX(t) = (Ax(t) + Bu(t))dt + \sigma(t)dW(t)$$

Now suppose we deal with an uncertain system from which only a rough model is available. Souto and do Val (n.d.) model the uncertainties regarding the operational model through extra noise terms dependending on the state and control action. In the scalar case, this leads to

$$dX(t) = (Ax(t) + Bu(t))dt + \sigma dW(t) + (\sigma^x + \overline{\sigma}^x |x(t)|)dW^x(t) + (\sigma^u + \overline{\sigma}^u |u(t)|)dW^u(t) \quad (4)$$

where A and B are scalars and $W^x(t), W^u(t)$ are one-dimensional Brownian motions.

As shown in Souto and do Val (n.d.), the performance of the controller on an interval $0 \leq s < T$, with T fixed, is measured by the expected value of a criterion of Bolza type

$$J(s, x, u(.)) := \mathbb{E}^x \left[\int_s^T f(t, x(t), u(t))dt + g(x(T)) \right]$$

Consider the optimal control problem over an infinite time horizon, a discounted quadratic running cost and assume the system matrices A, B as well as the drift coefficients σ, σ_u, $\overline{\sigma}_u$, σ_x, $\overline{\sigma}_x$ are time invariant. The linearized stochastic control problem can be formulated as

$$\min J(s, x, u) = \mathbb{E}^x \left[\int_s^\infty e^{-\rho t} \left(Q(x)^2 + H(u)^2 \right) \right] \quad (5)$$

$$\text{s.t. } dX(t) = (Ax(t) + Bu(t))dt + \hat{\sigma} d\hat{W}(t) \quad (6)$$

where

$$\hat{\sigma} d\hat{W}(t) = \sigma_t dW(t) + (\sigma^x + \overline{\sigma}^x |x_t|)dW_t^x + (\sigma_t^u + \overline{\sigma}_t^u |u_t|)dW_t^u \quad (7)$$

Moreover we consider $\sigma_x = 0$. Dynamic programming leads to the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} &-\frac{\partial \varphi}{\partial t} - \inf_{u \in U} H(t, x, u, \varphi_x, \varphi_{xx}) = 0 \\ &\varphi(T, x) = g(x) \end{aligned} \quad (8)$$

in which φ_x, φ_{xx} are the first and second order derivatives of $\varphi(T, x)$, and H the Hamiltonian function

$$\begin{aligned} H(t, x, u, \varphi_x, \varphi_{xx}) &= \frac{1}{2} \Big(\sigma^2 + \sigma_x^2 + 2\sigma_x \overline{\sigma}_x |x(t)| \\ &+ \overline{\sigma}_x^2 |x(t)|^2 + \sigma_u^2 + 2\sigma_u \overline{\sigma}_u |u(t)| + \overline{\sigma}_u^2 |u(t)|^2 \Big) \varphi_{xx} \\ &+ \varphi_x (Ax(t) + Bu(t)) + Qx(t)^2 + Ru(t)^2 \end{aligned} \quad (9)$$

The Hamiltonian function depends on the absolute values of the control $u(t)$ and state $x(t)$. It is therefore not differentiable, however, as shown in Souto et al. (2013) and Souto and do Val (n.d.), the optimal control sign can be determined. This leads to the division of the state space into three distinct regions $\mathcal{R}^+(t), \mathcal{R}^-(t)$ and $\mathcal{R}^0(t)$ in which the optimal control variation is greater, less than or equal to 0. The region $\mathcal{R}^0(t)$, in which the optimal control strategy is to keep the current control action unchanged, brings up the idea of a precautionary control and is known as the inaction region.

Inside the inaction region, the optimal control action in the linearized model is equal to 0 and the optimal cost function $V^*(x)$ is given by

$$V^*(x) = x^T X x + \frac{1}{\rho} \text{tr} \left(X \left(\sigma \sigma^T + \sigma_u \sigma_u^T \right) \right) \quad (10)$$

where X is the solution of the modified Lyapunov equation

$$A^T X + XA - \rho X + Q + \frac{1}{2} \text{Diag} \left(\overline{\sigma}_x^T X \overline{\sigma}_x \right) = 0 \quad (11)$$

In the scalar case, we assume the cost function in the inaction region has the form

$$V_0(x) = s_0 x^2 + p_0 x + l_0 \quad (12)$$

Replace $u = 0$ in (9) and solve the resulting HJB equation (8). This leads to

$$\begin{aligned} s_0 &= \frac{Q}{\rho - 2A - \overline{\sigma}_x^2} \\ p_0 &= 0 \\ l_0 &= \frac{(\sigma^2 + \sigma_u^2)}{\rho} s_0 \end{aligned} \quad (13)$$

We can also approximate the asymptotic behavior of the cost function. As stated in Souto and do Val (n.d.), if there is a solution to the modified Riccati equation

$$\begin{aligned} &A^T X + XA - \rho X - \\ &\qquad - XB \left(R' \right)^{-1} B^T X + Q' = 0 \\ &Q' := Q'(X) = Q + \overline{\sigma}_x^T X \overline{\sigma}_x \\ &R' := R'(X) = R + \overline{\sigma}_u^T X \overline{\sigma}_u \end{aligned} \quad (14)$$

then the value V^* tends asymptotically to $V_a^{\overline{s}}$

$$V_a^{\overline{s}} := x^T X x + \langle v(\overline{s}), x \rangle + l(\overline{s}) \quad (15)$$

with v and l satisfying

$$\begin{aligned}
&v\left(A-\rho I-2B\left(R'\right)^{-1}B^TX\right)= \\
&\qquad\qquad = 4\overline{s}^T\Delta_u(R')^{-1}B^TX \\
&l=\frac{1}{\rho}\left(\mathrm{tr}\left(X\left(\sigma\sigma^T+\sigma_u\sigma_u^T\right)\right)-\right. \\
&\qquad\left.2\left(\frac{1}{2}B^Tv+\Delta_u\overline{s}\right)^T\left(R'\right)^{-1}(\bullet)\right) \\
&\Delta_u=\Delta_u(X,\lambda):=\mathrm{Diag}\left(\overline{\sigma}_u^TX\overline{\sigma}_u\right)
\end{aligned} \tag{16}$$

In this region, the optimal control policy u^* tends to

$$u^*(x)=-K_\infty x+M_\infty \tag{17}$$

with $K_\infty = (R'(X))^{-1}B^TX$ and $M_\infty = (R'(X))^{-1}\left(\frac{1}{2}B^Tv+\Delta_u(X,\lambda)\overline{s}\right)$.

4 Optimal Harvesting

As shown in section 3, to use the strategy presented in Souto et al. (2013) we need a linearized model of the system to be controlled. Linearize the equations (1) and (2) around an equilibrium point (Z_e, h_e), substitute for the variations $x(t) = Z(t) - Z_e$ and $u(t) = h(t) - h_e$ and add the random terms of the CVIU approach. For the logistic function this leads to

$$dX(t)=\left(r\left(1-\frac{2Z_e}{K}\right)x(t)-u(t)\right)dt+\hat{\sigma}d\hat{W}(t) \tag{18}$$

For the Gompertz equation,

$$dX(t)=\left(\alpha\left[\log\left(\frac{K}{Z_e}\right)-1\right]x(t)-u(t)\right)+\hat{\sigma}d\hat{W}(t) \tag{19}$$

Adopt for the controller performance a quadratic discounted cost — suppose the operator wishes to minimize the control effort while striving to maintain the system around the equilibrium point — over an infinite time horizon. The optimal stochastic control problem can be stated as

$$\min\mathbb{E}\int_0^\infty e^{-\alpha t}\left(Qx(t)^2+Hu(t)^2\right)dt \tag{20}$$

$$\text{s.t.}\quad dX(t)=F(x(t),u(t))dt+\hat{\sigma}(t)d\hat{W}(t) \tag{21}$$

where $F(x(t), u(t))$ is replaced by (18) or (19) according to the chosen growth model.

With the corresponding CVIU models we can calculate a control strategy as presented in Section 3. In the scalar case, the optimal CVIU cost function is symmetric with respect to the origin. For the right half plane we have $u^* = 0$ and $V_0(x) = s_0x^2 + p_0x + l_0$ in the inaction region, which extends from $x = 0$ to the boundary point $x = x_b$. In a similar fashion, from a large enough x_∞ onward the asymptotic solution for the cost function (15) is valid and the optimal control action w.r.t. the CVIU problem is given by (17). Between these two points the optimal cost function is the solution of the nonlinear differential equation (8). An exact solution to this equation has not been found, therefore an approximation to the second derivative of the cost function and an order reduction are employed to solve equation (8) numerically in this interval. This procedure, as shown in Souto and do Val (n.d.), leads to residual values of the Hamilton-Jacobi-Bellman equation with relative error magnitude of 10^{-15}.

5 Numerical Computation

In this section we present numerical examples related to uncertainties such as those discussed in Section 1 and apply the CVIU controller to verify how it performs in harvest scenarios with large uncertainties. The performance of the proposed control strategy is then compared to the optimal LQG controller tuned to the estimated system dynamics. The actual system, as described by (1), is simulated through Monte Carlo evaluations with 50 repetitions and a time horizon of 200 seconds. The stochastic differential equation (1) is numerically computed by the Euler-Maruyama method as presented in Higham (2001). The harvested population is assumed to have a logistic (1) growth with parameters $K = 100$ and $r = 0.3$ operating around an equilibrium point $x_e = 3\frac{K}{4}$ as proposed in Roughgarden and Smith (1996).

We analyze how the system behaves when the parameters available to the controller differ from the ones of the actual system and in the presence of a stochastic disturbance given by a Brownian motion with diffusion coefficient σ. For that we, as proposed in Souto et al. (2013), evaluate the performance of the controller as the mismatch $\delta = A^0 - A$ between the original system parameter A^0 and the estimated value A increases. The relative gain of the CVIU control cost w.r.t. the LQG controller is shown in Figure 1 as the parameters $\overline{\sigma}_x$ and $\lambda_u = \frac{\sigma_u}{\overline{\sigma}_u}$ vary. Here a maximum gain of 13.75% is observed after the simulations.

We also simulate how fluctuations in the stock levels, modulated by the diffusion coefficient σ, impact on the controllers performance. For that we assume there is a 20% difference between the parameter A of the local linearized system and the estimate available to the controller. Results are shown in Figure 2. In both cases, $\sigma = 1$ or $\sigma = 2$, a maximum gain of about 14% of the CVIU controller is observed.

Following the arguments of Roughgarden and Smith (1996) and Sethi et al. (2005), we next evaluate how measurement errors affect the controlled harvest. The stock level assessment error was modeled as a random variable with uniform distribution $[1-\delta, 1+\delta]$ which is multiplied by the actual stock level. In Table 1 the relative gains of the CVIU approach when compared to the LQG

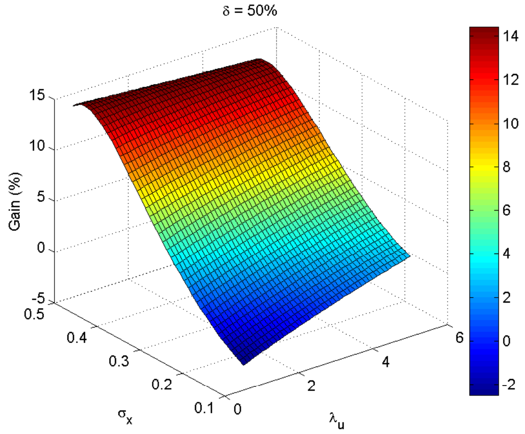


Figure 1: A 50% mismatch between the estimated parameter A and the actual system local approximation A^0.

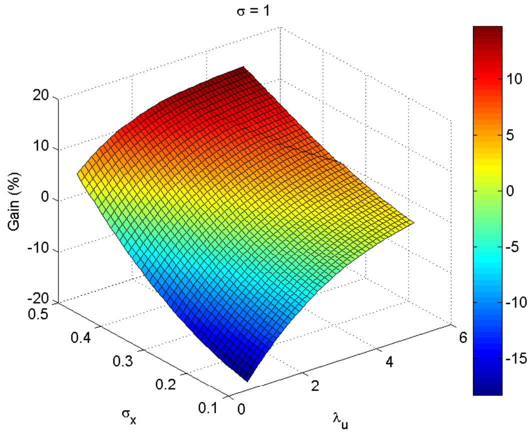
controller are shown for $\delta = 0.1$ and $\delta = 0.2$.

Table 1: Simulation results: CVIU gains (%) for stock level assessment errors

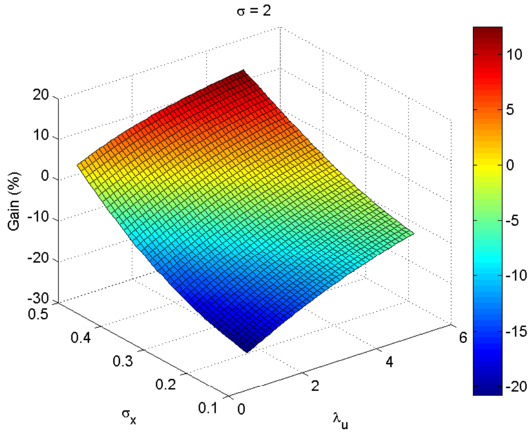
δ	$\overline{\sigma}_x$	min	mean	max
0.1	0.1	-18.27	-5.30	2.32
	0.2	-15.47	-3.0	4.20
	0.3	-10.91	0.72	6.85
	0.4	-4.35	5.76	10.80
	0.5	4.17	11.51	14.68
0.2	0.1	-17.95	-5.27	2.22
	0.2	-15.64	-3.01	4.21
	0.3	-10.53	0.81	6.69
	0.4	-4.47	5.74	10.55
	0.5	4.17	11.56	14.46

As presented in the works Sethi et al. (2005) and Roughgarden and Smith (1996), another uncertainty source consists of harvest implementation errors. Moreover, examples such as those of the Canadian cod collapse and the disagreements between different government agencies [Azevedo and Pierri (2014)] reinforce the perception that an evaluation of a possible mismatch between the attempted and the actual harvest is needed. Hence we simulate how the controlled system reacts to such disturbances. Similarly to the previous simulation, the attempted control action is multiplied bu a random variable with uniform distribution $[1-\delta, 1+\delta]$. Simulation results are shown in Table 2.

The numerical results presented in Tables 1 and 2 show the CVIU controller, when calibrated by the parameters $\overline{\sigma}_x$ and λ_u, performs better than the traditional LQG controller in the aforementioned scenarios, which were thought so as to mirror the uncertainties in the fisheries management literature.



(a)



(b)

Figure 2: Relative gains when $\sigma = 1$ (a) and $\sigma = 2$ (b).

6 Conclusion

In this paper we study how to manage renewable resources when there is limited knowledge about their dynamics. This is a topic of growing concern worldwide and, specially in the case of fisheries, uncertainties arise often in this class of problems, as pointed out by Roughgarden and Smith (1996), Sethi et al. (2005) and others.

The numerical results presented in Section 5 show that the CVIU approach yields better performance than the traditional LQG controller when large uncertainties affect the simulated system. Therefore we believe the CVIU approach can contribute to the study of fisheries management. Moreover, numerical computation presented in Souto and do Val (n.d.) show that the CVIU approach performs better than the LQG controller also in multidimensional systems. This let us speculate that the approach presented here can be extended to the harvesting problem of multi-species ecosystems.

In the harvest literature, a topic of concern is the evaluation of extinction risks of the har-

Table 2: Simulation results: CVIU gains (%) for implementation errors

δ	$\overline{\sigma}_x$	min	mean	max
0.1	0.1	-18.22	-5.41	2.27
	0.2	-15.68	-3.17	3.84
	0.3	-11.08	0.57	6.72
	0.4	-4.39	5.59	10.36
	0.5	3.98	11.32	14.66
0.2	0.1	-18.30	-5.73	1.62
	0.2	-15.91	-3.47	3.63
	0.3	-10.95	0.29	6.17
	0.4	-4.99	5.25	10.29
	0.5	3.32	10.89	13.93

vested populations, in order to avoid collapses such as that of the Canadian cod [Roughgarden and Smith (1996),Walters and Maguire (1996)]. Possible lines of future work include therefore the simulation of collapse risks when fisheries are under a CVIU controller and comparison with methods from the fisheries management literature.

Acknowledgments

This paper is based upon work supported by grant no. 2016/02208-6, São Paulo Research Foundation (FAPESP), Brazil.

References

Azevedo, N. and Pierri, N. (2014). A política pesqueira no brasil (2003-2011): a escolha pelo crescimento produtivo e o lugar da pesca artesanal, *Desenvolvimento e Meio Ambiente* **32**: 61–80.

Brock, W., Durlauf, S. and West, K. (2007). Model uncertainty and policy evaluation: Some theory and empirics, *Journal of Econometrics* **135**: 629–664.

Clark, C. W. and Kirkwood, G. P. (1996). On uncertain renewable stocks: Optimal harvest policies and the value of stock surveys, *Journal of Environmental Economics and Management* **13**(3): 235–244.

du Pin, A., Vallée, T. and do Val, J. (2009). Control variation as a source of uncertainty: single input case, *American Control Conference*.

FAO (2014). The state or world fisheries and acquaculture 2014, *Technical report*, Food and Agriculture Organization of the United Nations.

Higham, D. (2001). An algorithmic introduction to numerical simulation of stochastic differential equations, *SIAM Review* **43**(3): 252–246.

MMA (2006). Programa revizee: Avaliação do potencial sustentável de recursos vivos na zona econômica exclusiva - relatório executivo, *Technical report*, Secretaria de Qualidade Ambiental nos Assentamentos Humanos - Ministério do Meio Ambiente.

Murray, J. (2002). *Mathematical Biology*, Vol. I - An Introduction, third edn, Springer-Verlag.

Roughgarden, J. and Smith, F. (1996). Why fisheries collapse and what to do about it, *Proceedings of the National Academy of Sciences* **93**(10): 5078–5083.

Sethi, G., Costello, C., Fisher, A., Hanemann, M. and Karp, L. (2005). Fishery management under multiple uncertainty, *Journal of Environmental Economics and Management* **50**(2): 300–318.

Souto, R. (2015). *Modelagem e controle de sistemas estocásticos com dinâmica pouco conhecida*, PhD thesis, Faculdade de Engenharia Elétrica e de Computação - Universidade Estadual de Campinas.

Souto, R. and do Val, J. (n.d.). Modeling and control of stochastic systems with poorly known dynamics. Unpublished paper.

Souto, R., do Val, J. and Oliveira, R. (2013). Controlling uncertain stochastic systems: Performance comparisons in a scalar system, *IEEE Conference on Decision and Control*.

Walters, C. and Maguire, J.-J. (1996). Lessons for stock assessment from the northern cod collapse, *Reviews in Fish Biology and Fisheries* **6**(2): 125–137.