STOCHASTIC AUGMENTATION BY GENERALIZED MINIMUM VARIANCE CONTROL WITH RST LOOP-SHAPING

Tarcisio Carlos F. Pinheiro*, Anderson De F. Silva*, Antonio Da S. Silveira*, Maryson Da S. Araújo*

* *Laboratory of Control and Systems, Federal University of Pará.*
Belém, Pará, Brazil.
Post office box 479, 66075-110, R. Augusto Corrêa, 1 - Guamá, Belém-PA.

Emails: tpinheiro@ufpa.br, anso@ufpa.br, asilveira@ufpa.br, maryson@ufpa.br

Abstract— In this work we use the RST structure to shape the GMV optimization problem. The RST controller is tuned by pole assignment based on a second order plant model and a second order desired closed-loop model. The derived RST controller is then passed to the GMV generalized output weighting polynomials in order to produce a stochastic equivalent controller. The result is the equivalence of the RST and the GMV produced closed-loop dynamics whilst in ideal conditions (without noise or uncertainties), but a more economic and efficient GMV closed-loop dynamics under an adverse stochastic scenario.

1 Introduction

Control techniques to minimize costs associated to the maintenance and replacement of actuators are necessary to increase their lifespan. Actuators, such as solenoid valves, motor and other electromechanical or electro pneumatic actuators have a very important role to the process control area. Then, an appropriate control is necessary to improve the quality and reliability of such control loops. Thus, an effective control technique to minimize energy consumption and the variance of the controlled variable is necessary for control loop optimization. The Minimum Variance (MV) control is a way to achieve this goal using a stochastic approach (Silveira et al., 2016).

In this sense, what is uniquely investigated in this paper is how to do the stochastic augmentation of deterministic controllers. The method can augment any linear controller into a linear stochastic form that outperforms the deterministic one under the adverse scenario of noise and uncertainties, but keeps the same loop-shape of the deterministic control-loop. To the best of the authors knowledge, the stochastic augmentation by Minimum Variance is a new design procedure and being presented in this paper for the first time in the Americas and with a single already accepted article in an European conference occurring in this year of 2016, regarding the use of the stochastic augmentation by MV applied to power system stabilizers (reference omitted due to double-blind review requirements).

Åström introduced the Minimum Variance control and prediction problems to a wider audience during the decade of 1970 (Åström, 1970). He brought the MV concept to the process control industry, but the algorithm was limited due to energy constraints and nonminimum phase dynamics. Later, the Generalized Minimum Variance (GMV) algorithm was introduced by Clarke and Gawthrop (1975) to control nonminimum phase systems and with the possibility to minimize the control effor (Clarke, 1985).

Since its creation, the GMV technique is being applied to various fields of application, such as: medicine (Ganbing, 1988), aviation (Reynolds and Pachter, 1997), robotics (Shiino et al., 2008) and process control (Yamamoto et al., 1998). Despite its applicability, another interesting characteristic of GMV control is linked to its feature for interaction with other control structures (Mitsukura et al., 1999), such as the generalized RST controller (i.e. a digital controller tuned by pole placement design)(Åström and Wittenmark, 2013). This is what we exploit to do the stochastic augmentation.

The RST controller can be designed in the incremental form with characteristics of a dynamic compensator, as presented by Åström and Wittenmark (Åström and Wittenmark, 2013). This ensures that the RST system will have an observer part to avoid poles cancellations, therefore becoming more robust, similarly to the structure of a Linear Quadratic Gaussian compensator (despite not optimally tuned as well). What we do is to use this deterministic but robust RST system to do the loop-shaping of the GMV controller while using stochastic process design models at the same time. In the end, we have the same thing, but better, in the stochastic sense, due to active noise suppression characteristics inherited from the MV algorithm (Silveira et al., 2016).

The scope of this paper is limited to Single-Input, Single-Output linear systems and beyond this introduction it is organized as follows: in Section 2, the RST and GMV analogy is described; in Section 3, the incremental RST digital controller design is shown; in Section 4, GMV by RST loop shaping is presented; in Section 5, the robustness analysis is covered; in Section 6, main simulation results are presented using an stochastic process; the paper finishes with Conclusions and future works.

2 RST and GMV analogy

The GMV optimization problem is solved based on the generalized output (Clarke and Gawthrop, 1975),

$$\phi(k+d) = P(z^{-1})y(k+d) - T(z^{-1}) \\ y_r(k+d) + Q(z^{-1})\Delta u(k) \quad (1)$$

described in the backward-shift operator domain, z^{-1}, where:

$$\begin{aligned} P(z^{-1}) &= 1 + p_1 z^{-1} + \cdots + p_{n_p} z^{-n_p} \\ T(z^{-1}) &= t_0 + t_1 z^{-1} + \cdots + t_{n_t} z^{-n_t} \\ Q(z^{-1}) &= q_0 + q_1 z^{-1} + \cdots + q_{n_q} z^{-n_q} \end{aligned} \quad (2)$$

By considering a type-0 AutoRegressive-Moving-Average with eXogenous inputs (ARMAX) system, the GMV problem is augmented with incremental control action for guaranteed servo tracking, which is,

$$u(k) = u(k-1) + \Delta u(k) \quad (3)$$

with $\Delta = 1 - z^{-1}$.
If there exists a u(k) that stabilizes the ARMAX system, then $\phi(\infty) \to 0$. From this time on, the generalized output in (1) resembles the RST structure. GMV and RST are respectively presented as:

$$Q(z^{-1})\Delta u(k) = T(z^{-1})y_r(k+d) - Py(k+d) \quad (4)$$

$$R(z^{-1})\Delta u(k) = T_{RST}(z^{-1})y_r(k+d) - Sy(k) \quad (5)$$

It means that the GMV controller will take care of the minimization of the variance of $\phi(k)$, i.e. the minimization of σ_ϕ^2, subjected to the RST constraint imposed by $Q(z_{-1}), P(z_{-1}), T(z_{-1})$ weighting polynomials by using the following equivalence:

$$\begin{aligned} Q(z^{-1}) &:= R(z^{-1}) \\ P(z^{-1}) &:= S(z^{-1}) \\ T(z^{-1}) &:= T_{RST}(z^{-1}) \end{aligned} \quad (6)$$

where (Åström and Wittenmark, 2013),

$$\begin{aligned} R(z^{-1}) &= 1 + r_1 z^{-1} + r_2 z^{-2} + \cdots + r_n z^{-n} \\ S(z^{-1}) &= s_0 + s_1 z^{-1} + s_2 z^{-2} + \cdots + s_n z^{-n} \\ T(z^{-1}) &= t_0 + t_1 z^{-1} + t_2 z^{-2} + \cdots + t_n z^{-n} \end{aligned} \quad (7)$$

By RST pole placement design based on prescribed closed-loop dynamics, one can use the polynomials of (4) into (3). The GMV controller will then exhibit the same I/O behavior as the RST but with predictive characteristics, since the reference and the output signals are used d-steps ahead. In order to have a fairest comparison setup, one may test the GMV case without the d-steps ahead reference sequence $y_r(k)$. Notice, however, that if the same is done to $y(k+d)$, the controller will no longer produce the minimization of σ_ϕ^2 in the minimum variance sense.

3 Incremental RST digital controller

The ARMAX plant model is considered,

$$A(z^{-1})y(k) = B(z^{-1})u(k-d) + C(z^{-1})\xi(k) \quad (8)$$

where,

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} \\ C(z^{-1}) &= 1 + c_1 z^{-1} + c_2 z^{-2} \end{aligned} \quad (9)$$

The augmented design model considered is in the Autoregressive Integrated moving average model with exogenous inputs (ARIMAX) form,

$$\Delta Ay(k) = B(z^{-1})\Delta u(k-d) + C(z^{-1})\xi(k) \quad (10)$$

where, $A(z^{-1}) = 1 + \bar{a}_1 z^{-1} + \bar{a}_2 z^{-2} + \bar{a}_3 z^{-3}$ and $\xi(k)$ is a white-noise sequence. In Figure 1 it is shown the block diagram of the RST control loop and the deterministic part of the plant model. The disturbance on system is represented by $d_o(k)$.

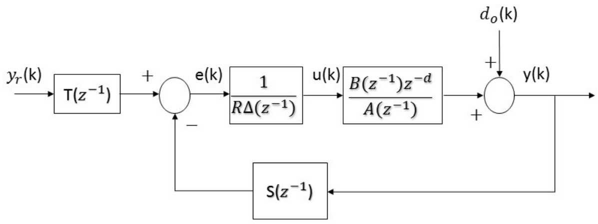

Figure 1: Block diagram of RST control.

$R(z^{-1}), S(z^{-1})$ and $T(z^{-1})$, shown in (7) are obtained by solving the following pole assignment Diophantine equation (Åström and Wittenmark, 2013):

$$H_{MF}(z^{-1}) = A(z^{-1})\Delta R(z^{-1}) + z^{-d} \\ B(z^{-1})S(z^{-1}) \quad (11)$$

where,

$$H_{MF}(z^{-1}) = H_c(z^{-1})H_0(z^{-1}) \quad (12)$$

In (12), $H_0(z^{-1}) = A(z^{-1})$ is an observable part. It guarantees that the open-loop poles are preserved (not canceled). $H_c(z^{-1})$ is the desired characteristic polynomial that in this paper is of second order. The closed-loop desired convergence will then be damped or under-damped, as desired.

$H_{MF}(z^{-1})$ will be monic and its order will be 4, considered as

$$H_{MF}(z^{-1}) = 1 + h_1 z^{-1} + h_2 z^{-2} \\ + h_3 z^{-3} + h_4 z^{-4} \quad (13)$$

One solution to the Diophantine equation in (5) is given by:

$$R(z^{-1}) = 1 + r_1 z^{-1} \quad (14)$$

$$S(z^{-1}) = s_0 + s_1 z^{-1} + s_2 z^{-2} \quad (15)$$

$$T_{RST}(z^{-1}) = t_0 + t_1 z^{-1} + t_2 z^{-2} \quad (16)$$

where,

$$\begin{aligned} r_1 &= \frac{h_4}{\bar{a}_3} \\ s_0 &= \frac{h_1 - \bar{a}_1 + r_1}{B(1)} \\ s_1 &= \frac{h_2 - \bar{a}_2 + r_1\bar{a}_1}{B(1)} \\ s_2 &= \frac{h_3 - \bar{a}_3 + r_1\bar{a}_2}{B(1)} \end{aligned} \quad (17)$$

with B(1) being an static approximation to simplify the Diophantine solution. The polynomial $T(z^{-1})$ is computed to guarantee an offset free closed-loop system, such that

$$T(z^{-1}) = toffH_0(z^{-1}) \quad (18)$$

The toff parameter is created to pre-compensate through a static approximation of the error between reference and output of the plant (Åström and Wittenmark, 2013). The toff is given by:

$$toff = \frac{H_c(1)}{B(1)} \quad (19)$$

4 GMV by RST loop-shaping

To design the GMV controller it is necessary to find the polynomials $F(z^{-1})$ and $E(z^{-1})$ through the following Diophantine equation (Clarke and Gawthrop, 1975):

$$\begin{aligned} P(z^{-1})C(z^{-1}) = \Delta A(z^{-1})E(z^{-1}) \\ +z^{-1}F(z^{-1}) \end{aligned} \quad (20)$$

According to Clarke and Gawthrop (1975), $F(z^{-1})$ and $E(z^{-1})$ polynomials can be obtained by:

$$\begin{aligned} &F(z^{-1}) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots + f_{nf} z^{-nf} \\ &n_f = max(np + nc - 1, na - 1); \\ &E(z^{-1}) = e_0 + e_1 z^{-1} + e_2 z^{-2} \cdots + e_{nf} z^{-ne} \\ &n_e = d - 1; \end{aligned} \quad (21)$$

Only d=1 is considered in this paper, then one solution to the Diophantine equation in (20) is given by:

$$E(z^{-1}) = e_0 \quad (22)$$

$$F(z^{-1}) = f_0 + f_1 z^{-1} + f_2 z^{-2} + f_3 z^{-3} \quad (23)$$

where,

$$\begin{aligned} e_0 &= s_0 \\ f_0 &= s_0 c_1 + e_0 - a_1 e_0 \\ f_1 &= s_0 c_2 + s_1 c_1 + s_2 - e_0 a_1 - a_2 e_0 \\ f_2 &= s_1 c_2 + s_2 c_1 + a_2 e_0 \\ f_3 &= s_2 c_2 \end{aligned} \quad (24)$$

In Figure 2 it is shown the block diagram of the GMV controller with the stochastic plant model.

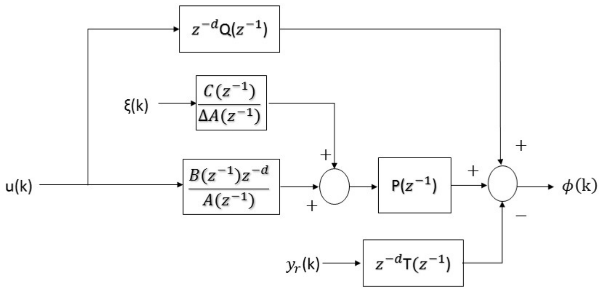

Figure 2: GMV control loop block diagram.

The GMV incremental control law, is given by (Clarke and Gawthrop, 1975):

$$\Delta u(k) = \frac{C(z^{-1})T(z^{-1})y_r(k+d) - F(z^{-1})y(k)}{B(z^{-1})E(z^{-1}) + C(z^{-1})Q(z^{-1})} \quad (25)$$

5 Robustness analysis

Robustness analysis using sensitivity functions (magnitude plots in the frequency domain) is done in this section. From these functions, important indices can be obtained to quantify the trade-off between robustness and performance (Seborg et al., 2010).

The sensitivity function, $S_{sen}(e^{jwT_s})$, in (26) is affected by a disturbance, for example, $\xi(k)$. Moreover, the complementary sensitivity function, $T_{com}(e^{jwT_s})$, in (27) is affected by a reference signal, $y_r(k)$.

$$S_{sen}(e^{jwT_s}) = \frac{1}{1 + G_c(z^{-1})G_p(z^{-1})} \quad (26)$$

$$T_{com}(e^{jwT_s}) = \frac{G_c(z^{-1})G_p(z^{-1})}{1 + G_c(z^{-1})G_p(z^{-1})} \quad (27)$$

The complete sensitivity equation is given by:

$$y(k) = T_{com}(e^{jwT_s})y_r(k) + S_{sen}(e^{jwT_s})\xi(k) \quad (28)$$

$G_p(z^-1)$ is the plant model and $G_c(z^-1)$ is the controller, RST as in (5) or GMV as in (25). The closed-loop transfer function of the RST control loop can be obtained by substituting (5) into (10) and the complementary sensitivity and sensitivity functions are given by

$$y(k) = \frac{B(z^{-1})T(z^{-1})}{B(z^{-1})S(z^{-1}) + A(z^{-1})\Delta R(z^{-1})} y_r(k+d) + \frac{C(z^{-1})R(z^{-1})}{B(z^{-1})S(z^{-1}) + A(z^{-1})\Delta R(z^{-1})}\xi(k) \quad (29)$$

while the complementary sensitivity and sensitivity functions of the GMV can be obtained by substituting (25) into (10), resulting in

$$y(k) = \frac{B(z^{-1})T(z^{-1})}{B(z^{-1})P(z^{-1}) + A(z^{-1})\Delta Q(z^{-1})} y_r(k+d) + \frac{B(z^{-1})E(z^{-1}) + C(z^{-1})Q(z^{-1})}{B(z^{-1})P(z^{-1}) + A(z^{-1})\Delta Q(z^{-1})} \xi(k) \quad (30)$$

One way to quantify the sensibility of the control loop system is through the maximum amplitude ratios of the complementary and sensitivity functions, respective to the peaks M_S and M_T. Small M_s values makes the system less sensible to disturbance. Furthermore, M_T is equivalent to the amplitude of the resonant peak as well, that in general, is desirable to be kept small (Seborg et al., 2010).

Desirable values for M_S and M_T are, respectively, in the range of 1.2-2 and 1-1.5, giving balance between robustness and performance (Seborg et al., 2010). These two indices are:

$$M_S = max|S(e^{jwT_s})| \quad (31)$$

$$M_T = max|T(e^{jwT_s})| \quad (32)$$

To obtain equilibrium between robustness and performance it is suggested a range of 4.6 dB-12dB and 30° to 45°, for gain and phase margins, respec-tively, GM and PM (Seborg et al., 2010). These indices can be obtained by the following equations:

$$GM_S \geq \frac{M_S}{M_S - 1} \quad (33)$$

$$PM_S \geq 2sin^{-1}\left(\frac{1}{2M_S}\right)\left(\frac{180}{\pi}\right) \quad (34)$$

$$GM_T \geq 1 + \frac{1}{M_T} \quad (35)$$

$$PM_T \geq 2sin^{-1}\left(\frac{1}{2M_T}\right)\left(\frac{180}{\pi}\right) \quad (36)$$

6 Simulation and Performance

In this section, a DC motor is the plant model used for simulations. Which has a continuous-time trans-fer function as

$$G(s) = \frac{k}{s(\tau s + 1)} \quad (37)$$

where the control signal is input voltage to the motor and the output is the angular position. The discrete-time model with zero-order-hold was obtained through recursive least squares identification, with sampling time of 0.05 s. The Fig. 3 shows the comparison between real model and ARMAX model identified. The ARMAX polynomials are:

$$\begin{aligned} A(z^{-1}) &= 1 - 0.1718z^{-1} - 0.8282^{-2} \\ B(z^{-1}) &= 0.05352 - 0.07687^{-1} \\ C(z^{-1}) &= 1 + 0.378^{-1} - 0.1688z^{-2} \end{aligned} \quad (38)$$

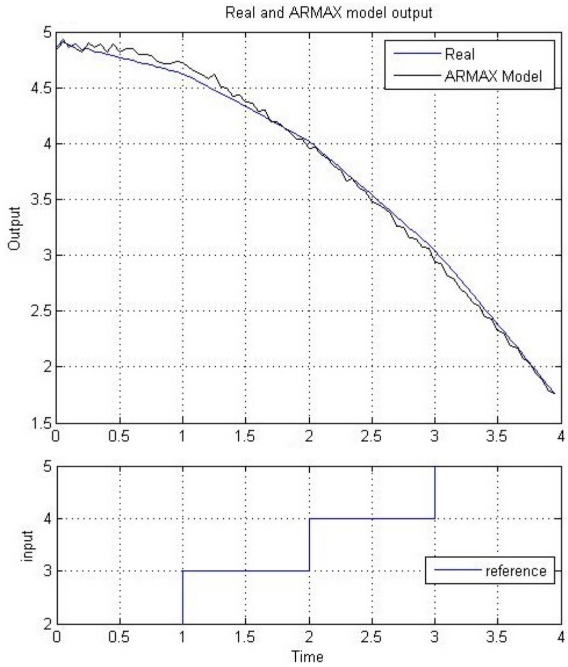

Figure 3: Real and ARMAX Model output.

The performance of the GMV controller with RST loop shaping can be evaluated by a discrete approximation of the integral of a squared signal, given by

$$ISW = \sum (w^T w)T_s \quad (39)$$

where w is a general vector that may assume the input, the output, the tracking error and the generalized output (Silveira et al., 2016). It is used in this paper to evaluate how much power is consumed by the controller in order to keep the output variance as minimal as possible. For simulation test was used variance of the white-noise sequence of 0.01.

The graphical result about GMV and RST controllers are presented in Fig. 4, Fig. 5, and Fig. 6. From these results, variances and power indices ISW were obtained for reference tracking, regulatory simulation, and step load simulation test for GMV and RST controllers to the system output and control signal. All these performance indices are presented within the titles of figures 4 and 5 and 6.

Analyzing the results between controller performance, the signal variance, generalized output σ^2_ϕ and ISW of the output and control signal, using the GMV controller those indices are smaller than the values obtained using the RST controller. Then the GMV controller consumes less power to do the same thing but even better. This result is evident since the loop-shape is the same. It is because the GMV sensitivity function shown in (30) has two polynomials, $B(z^{-1})$ and $E(z^{-1})$, that do not exist in the RST sensitivity function in (29) and this difference is responsible for a controller that mimics the loop-shape but will reduce the power consumption and increase de output quality.

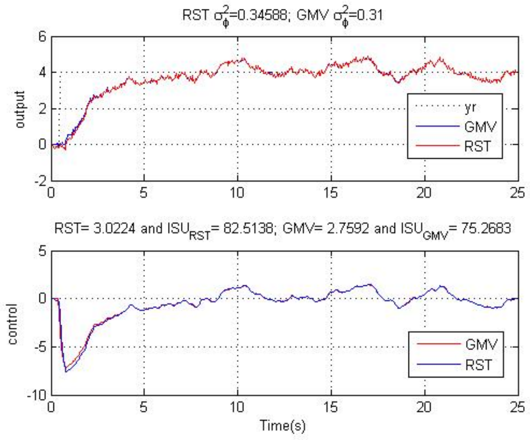

Figure 4: Simulation Results RST-GMV to reference tracking

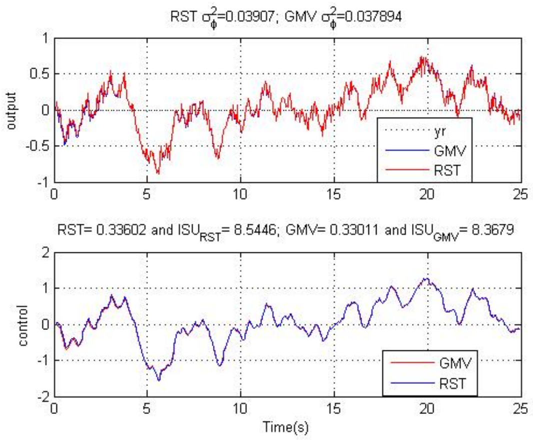

Figure 5: Regulatory simulation test.

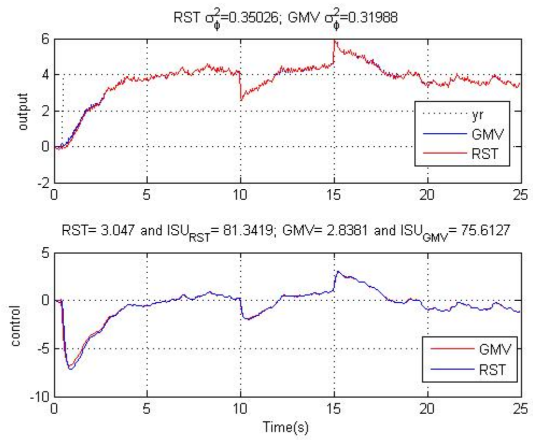

Figure 6: Step load simulation test.

The robustness index obtained from (33) to (36) are shown in Table 1. Analyzing this table, together with the Bode diagram shown in Figure 7, the phase margin is PM $= 60°$, characteristic of a robust control system (Anderson and Moore, 2007).

Table 1: Robustness indices results

	RST	**GMV**
GM	6dB	6dB
PM	60°	60°

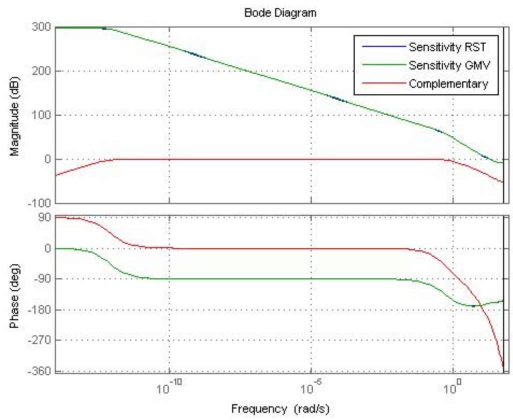

Figure 7: Bode diagram of the RST and GMV control loops.

7 Conclusions

A GMV control algorithm with RST loop-shaping was presented and its application to improve performance has been shown, depicting in a short format what we have been presenting as stochastic augmentation of deterministic controllers. This technique results, mainly those shown at figures 4 and 5 and 6, seem to be an effective method for minimize the control signal variance, consequently, it makes the control signal more economic and efficient by still doing the same deterministic job as before. It is obvious due to the fact that the complementary sensitivity function, or simply the deterministic closed-loop transfer function, is exact the same.

The stochastic augmentation procedure then focuses on how to minimize the variance from the control signal and output system, but maintaining the loop-shape. However, our initial thought was to use a GMV controller to interact with other control structures, for example, to tune PID controllers as is commonly seen in the literature, but after some re-cent results presented in Silveira et al. (2016) become available, we saw the possibility of the method presented here in this paper.

The RST controller was chosen because robustness stability would be guaranteed by design, by simple selection of a first order system as the closed-loop transfer function. Then, the parameters from this RST synthesis would be transferred to the GMV controller. However, any linear controller can be used. The only requirement is that the ARMAX system must be identified by some regression method in order to model the stochastic system and quantify the process uncertainties as a Gaussian disturbance of a known variance or at least a worst case variance.

References

Anderson, B. D. and Moore, J. B. (2007). *Optimal control: linear quadratic methods*, Courier Corporation.

Åström, K. J. (1970). Introduction to stochastic control theory, *Richard Bellman, University of Southern California* .

Åström, K. J. and Wittenmark, B. (2013). *Computer-controlled systems: theory and design*, Courier Corporation.

Clarke, D. (1985). Introduction to self-tuning controllers, *IEE control engineering series* **15**: 36–71.

Clarke, D. W. and Gawthrop, P. J. (1975). Self-tuning controller, *Electrical Engineers, Proceedings of the Institution of* **122**(9): 929–934.

Ganbing, W. (1988). Adaptive controller of blood pressure using model with fraction time delays, *Proceedings of the Annual International Conference of the IEEE Engineering in Medicine and Biology Society*.

Mitsukura, Y., Yamamoto, T. and Kaneda, M. (1999). A design of self-tuning pid controllers using a genetic algorithm, *American Control Conference, 1999. Proceedings of the 1999*, Vol. 2, IEEE, pp. 1361–1365.

Reynolds, O. and Pachter, M. (1997). The performance of generalized minimum variance system identification, *Proc. Fifth IEEE Mediterranean Conf. on Control and Systems*.

Seborg, D. E., Mellichamp, D. A., Edgar, T. F. and Doyle III, F. J. (2010). *Process dynamics and control*, John Wiley & Sons.

Shiino, T., Kawada, K., Yamamoto, T., Komichi, M. and Nishioka, T. (2008). Gimbals control with the camera for aerial photography in rc helicopter, *Control, Automation and Systems, 2008. ICCAS 2008. International Conference on*, IEEE, pp. 1135–1139.

Silveira, A., Trentini, R., Coelho, A., Kutzner, R. and Hofmann, L. (2016). Generalized minimum variance control under long-range prediction horizon setups, *ISA transactions* **62**: 325–332.

Yamamoto, T., Fujii, K. and Kaneda, M. (1998). Self-tuning temperature control of a polymerizing reactor, *Control Applications, 1998. Proceedings of the 1998 IEEE International Conference on*, Vol. 2, IEEE, pp. 1110–1114.