RELAY FEEDBACK METHOD FOR A QUADROTOR ALTITUDE CONTROL

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Abstract— This paper presents the development of an altitude controller for a low cost quadrotor. In this work, an empirical tuning method of PID controllers is employed, whose great benefit is that it does not need mathematical model. In addition, the relay method is used in the estimation of tuning parameters and there are not any studies available in the literature about the application of this method for quadrotor altitude control. The design methodology, which is suitable for low-cost vehicles whose modelling is difficult to obtain, is described in the paper as well as the referred experiments. In the experiments, the following performance parameters are evaluated on transient state for a step reference input: overshoot, rise time and settling time. Furthermore, the response of the altitude for a reference trapezoidal trajectory is evaluated. The results show that the technique is simple and efficient for this kind of system.

Keywords— Relay Method, Altitude Control, Unmanned Aerial Vehicle, Quadrotor, Empirical Tuning.

Resumo— Neste artigo é apresentado o desenvolvimento de um controlador de altitude para um veículo quadrirrotor de baixo custo. Neste trabalho, é aplicada uma metodologia de sintonia empírica de controladores PID que apresenta como grande vantagem a dispensa de modelo matemático. Além disso é utilizado o método do relé na estimação dos parâmetros de sintonia, e não se tem estudos disponíveis na literatura de sua aplicação em sistemas de controle de altura de quadrirrotores. A metodologia de projeto, adequada para veículos de baixo custo cuja modelagem é de difícil obtenção, é descrita no trabalho, assim como os experimentos do sistema em questão. Nos experimentos foram avaliados os seguintes parâmetros de desempenho em regime transitório para uma entrada de referência do tipo degrau: sobre sinal, tempo de subida e tempo de acomodação. Além disso, foi avaliado o comportamento da altitude compensada para uma trajetória de referência trapezoidal. Os resultados mostraram que a técnica empregada é, além de simples, eficiente para este tipo de sistema.

Palavras-chave— Método do Relé, Controle de Altitude,Veículos Aéreos Não Tripulados, Quadrirrotor, Sintonia Empírica.

1 Introduction

Autonomous control of unmanned aerial vehicles (UAVs) have received considerable attention in the last decades primarily driven by an increasing commercial and military exploration of these robots. Particularly, the quadrotor vehicles aroused much interest because of their ability to take off and landing vertically, great mobility, high load capacity, simplicity and low costs compared to other similar UAVs.

Recently, development of autonomous UAV systems with less-human interaction have been encouraged. Particularly, there are three levels of autonomy for UAV systems: autonomous navigation, autonomous landing, and autonomous taking-off. For these different autonomy purposes, different control problems and control subsystems rise, *e.g.* stability, attitude control, altitude control, and position control. Specially, the altitude control is an essential component for any autonomous UAV systems.

The quadrotor control is a challenging problem, mainly due to the fact that it is an underactuated system and its model has many nonlinear characteristics, specially if the complex aerodynamic effects are considered. One of the challenges in designing these control systems is the difficulty in obtaining a precise parametric model of these systems. An alternative to avoid this problem is the use of empirical techniques for control design (*e.g.* Ziegler-Nichols tuning and Tyreus-Luyben tuning) that dispenses a mathematical model.

Different strategies have been proposed to control attitude, altitude, and position of UAVs systems, most of them based in complex mathematical models. Among the different proposals, there may be mentioned: classical PID control (Bouabdallah et al., 2004), robust control approaches (Kun and Hwang, 2016) , optimal techniques (Lai et al., 2006), adaptive control (Alexis et al., 2012), back-stepping (Nagaty et al., 2013), and sliding mode (Xue et al., 2016; Xiong and Zhang, 2016).

There is a lack of applications of empirical tuning techniques in UAV control design, those techniques are extensively employed in industrial process due to their efficiency, *i.e.* empirical controllers offer a reasonable performance dispensing a process model description. One of the few works that employs empirical tuning for UAV control is presented by (Julkananusart and

Nilkhamhang, 2015), where Fictitious Reference Iterative Tuning is used for attitude control of a quadrotor system. In (Yang et al., 2013), an adaptive self-tuning approach is used for a PID control strategy for attitude control of a quadrotor vehicle.

In tuning techniques like Ziegler-Nichols (ZN) and Tyreus-Luyben (TL), is demanded some information about the systems, that might be obtained experimentally, such as critical gain and critical period. A well-known methodology for auto-tuning PID controllers is the Åström-Hägglund method, also called relay method, presented in (Åström and Hägglund, 1984). This method provides an adequate estimation of these parameters without a risk of loop instability. This methodology is usually employed in industrial process, where the mathematical models are hardly obtained, like in (Dandan et al., 2014).

The main objective of this research is the implementation and comparison of altitude control strategies for a low-cost quadrotor UAV systems without using of physical nor parametric models. For this purpose, we proposed a empirical controller. This controller is designed with two different tuning approaches: Ziegler-Nichols (ZN) and Tyreus-Luyben (TL). The critical gain and period demanded are obtained using the relay method. To the best of our knowledge, the relay tuning method was never used before for controlling UAV systems. These strategies are implemented in a real low-cost quadrotor and their performances were compared.

The remainder of the article is organized as follows: Section 2 describes the UAV quadrotor model, Section 3 explains the controller design, Section 4 describes the experimental results, and Section 5 presents the conclusions and describes future research.

2 Modelling of the quadrotor UAV system

The vehicle body frame B and the earth fixed frame E are illustrated in Fig. 1, where B is used to represent row (ϕ), pitch (θ) and yaw (ψ) motions and E is used to represent x, y and z translational motions in the three dimensional space. The B to E mapping is performed by the rotation matrix $\mathbf{R}$, where $c_{(\cdot)} = \cos(\cdot)$ and $s_{(\cdot)} = \sin(\cdot)$.

$$\mathbf{R} = \begin{bmatrix} c_\psi c_\theta - s_\phi s_\psi s_\theta & -c_\phi s_\psi & c_\psi s_\theta + c_\theta s_\phi s_\psi \\ c_\theta s_\psi - c_\psi s_\phi s_\theta & c_\phi c_\psi & s_\psi s_\theta - c_\psi c_\theta s_\phi \\ -c_\phi s_\theta & s_\phi & c_\phi c_\theta \end{bmatrix} \quad (1)$$

The attitude and position models are shown in Eq. 2 (Bouabdallah et al., 2004), where I_{xx}, I_{yy} and I_{zz} are the moments of inertia around B axis, m is the mass of the quadrotor, g is gravitational acceleration, U_i corresponds to the thrust forces

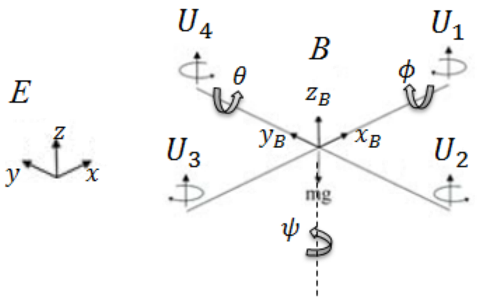

Figure 1: Body fixed frame B and earth fixed frame E representations. Adapted from (Bouabdallah et al., 2004).

produced by the propellers and U_t is the thrust force resultant.

$$\begin{aligned} \ddot{\phi} &= \frac{I_{yy} - I_{zz}}{I_{xx}} \dot{\psi}\dot{\theta} + \frac{u_x}{I_{xx}} \\ \ddot{\theta} &= \frac{I_{zz} - I_{xx}}{I_{yy}} \dot{\psi}\dot{\phi} + \frac{u_y}{I_{yy}} \\ \ddot{\psi} &= \frac{I_{xx} - I_{yy}}{I_{zz}} \dot{\phi}\dot{\theta} + \frac{u_z}{I_{zz}} \\ \ddot{x} &= [c_\phi s_\theta c_\psi + s_\phi s_\psi] + \frac{U_t}{m} \\ \ddot{y} &= [c_\phi s_\theta s_\psi - s_\phi c_\psi] + \frac{U_t}{m} \\ \ddot{z} &= -g + [c_\phi c_\theta] \frac{U_t}{m} \end{aligned} \quad (2)$$

The u_x, u_y e u_z torques are given by Eq. 3, where l is the distance from the axis of rotation of the rotors to the center of the quadrotor and M_i are the moments generated by the rotation of the propellers.

$$\begin{aligned} u_x &= l(U_2 - U_4) \\ u_y &= l(U_1 - U_3) \\ u_z &= (M_1 + M_3) - (M_2 + M_4) \end{aligned} \quad (3)$$

The model presented in Eq. 2 requires the knowledge of physical parameters. As these parameters of the quadrotor used in this paper are unknown, it is necessary to use empirical methods to design the altitude controller.

3 Altitude controller design

The altitude structure control is illustrated in Fig. 2. The gains of the PI controller was performed using Astrom method, depicted in Fig. 3, known as the ideal relay method (Åström and Hägglund, 1984). In quadrotor altitude control we used an conditional integration anti windup technique (Peng et al., 1996), since the control signal saturation occurs and the integral term tends to become large. Thus the comparator block switches the integral input according to u and ur signals.

The quadrotor input is a PWM signal. The relation between ur and pwm is given by Eq. 4. The pwm_{ref} is a PWM value for the thrust performed by the propellers is equal the weight of quadrotor.

$$pwm = pwm_{ref} + ur \tag{4}$$

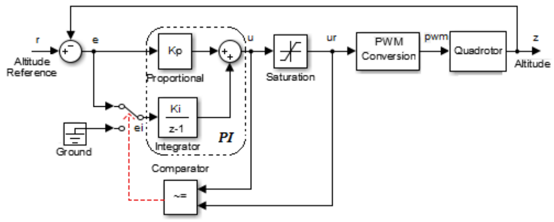

Figure 2: Control structure diagram.

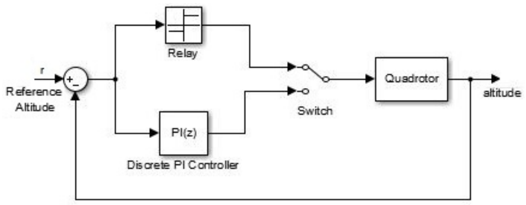

Figure 3: Ideal relay method scheme.

The relay method idea is to create a variation in the output process, which based on the frequency response, the critical period T_u and the critical gain K_u are estimated.

The process oscillation is controlled by the relay amplitude h, illustrated in Fig. 4. The estimated critical gain is determined by Eq. 5.

$$K_u = \frac{4h}{\pi a} \tag{5}$$

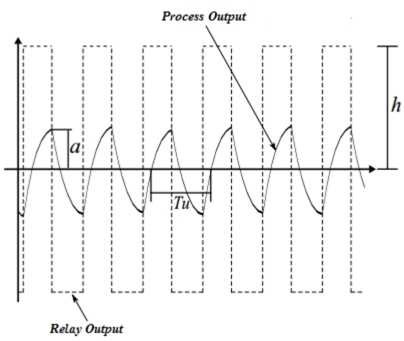

Figure 4: Process output for relay method. Adapted from (Pinto, 2014).

The altitude response using the relay method is illustrated in Fig. 5. The relay amplitude represent the PWM variation of the process input. The parameters obtained are shown in Tab. 1.

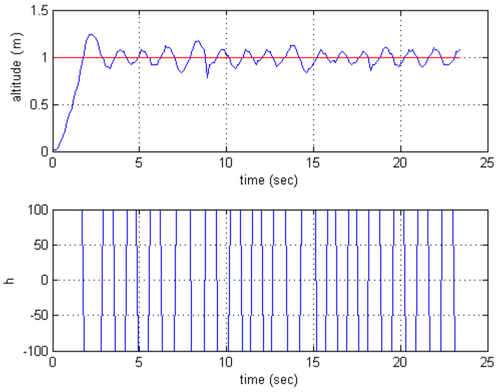

Figure 5: Altitude response for relay input.

Table 1: Relay method parameters.

Parameter	Value
h	100
a	0.06 (m)
T_u	1.2 (sec)
K_u	2122

Ziegler-Nichols (ZN) or Tyreus-Luyblen (TL) rules are used to obtain the PI gains according to Tab. 2 and Tab. 3, respectively. The PI controller is digital so it is necessary to consider the sample time T_s. Thus the discrete integral controller gain K_i is given by Eq. 6 for ZN rules. For TL rules the same gain is given by the product of K_i and T_s.

Table 2: ZN tuning rules (Ziegler and Nichols, 1942).

Controller	K_p	T_i	T_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$T_u/1.2$	-
PID	$0.6K_u$	$T_u/2$	$T_u/8$

$$K_i = \frac{K_p.T_s}{T_i} \tag{6}$$

4 Experimental Evaluation

4.1 Experimental Methodology and Environment

This section presents the experimental results of the altitude control. The experiments were performed considering the test structure shown in Fig. 6. The structure has 2.5 meters of height and 1.25 meters x 1.25 meters of base and has two cylindrical steel bars with 2.5 meters x 0.5 inch separated 27 centimeters from each other. This structure was constructed to allow only vertical movement.

The quadrotor model used in this paper, shown in Fig. 7, has a x600 model frame with 60 cm of wingspan, one Multiwii Pro board with an

Table 3: TL tuning rules (Al-Younes, 2009)

Controller	K_p	T_i	T_d
PI	$0.3125K_u$	$0.142\frac{K_u}{T_u}$	-
PID	$0.45K_u$	$0.206\frac{K_u}{T_u}$	$0.075K_uT_u$

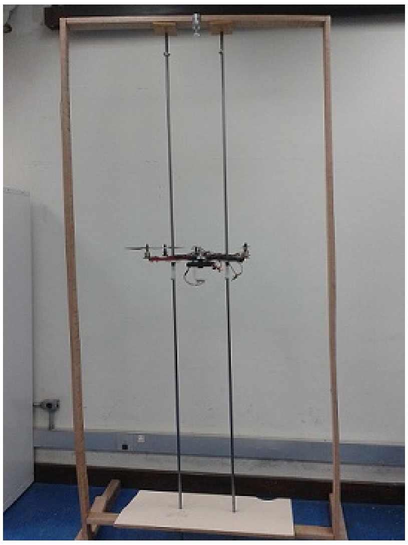

Figure 6: Test structure.

Atmega 2560 microcontroller, four brushless DC motors A2212 / 13T of 1000KV, four ESCs (Electronic Speed Controller) of 20A, a LiPo battery of 3500mAh 11.1 volts (3S), a pair of 9 x 4.7 SF (Slow Flyer) propellers and another pair of 9 x 4.7 SFP (Slow Flyer Propeller) propellers. Furthermore, quadrotor has a HC-SR04 ultrasonic sensor disposed at the center of the vehicle to measure the altitude. The vehicle mass is 1.2Kg.

Figure 7: Quadrotor model used.

The experimental evaluation of the altitude response was made considering the tuning method of the controller, the control signal saturation and the performance parameters for a step reference input. The performance parameters were the percent overshoot ($P.O$), rise time (t_r) and settling time (t_s) measured as (Ogata, 2010). In all tests the reference value of the PWM signals applied to all ESCs was 1525us and the sample time was 0.1s for altitude reading.

4.2 Experimental Results

In Fig. 8 are shown the altitude responses with PI gains obtained from the Ziegler-Nichols (ZN) tuning rules considering the control signal saturation at 100, 150 and 200. Percent overshoot, rise time and settling time are shown in Tab. 4.

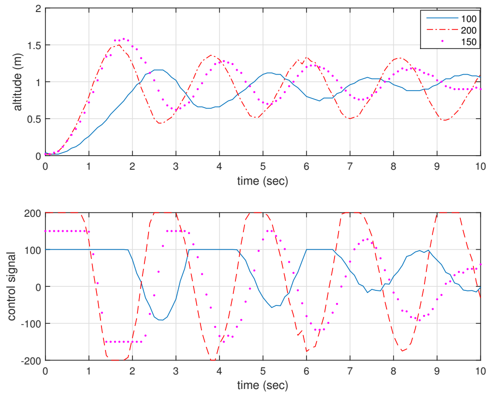

Figure 8: Altitude comparative graphs for PI controller using the ZN gains: $K_p = 955$ and $K_i = 95$. Altitude reference = 1.0 m.

Table 4: Performance parameters measured using three values of saturation. PI tuning gains: $K_p = 955$ $K_i = 95$.

Saturation	$P.O(\%)$	t_r(sec)	t_s(sec)
100	16	1.6	>10
150	58	0.8	>10
200	50	0.7	>10

The altitude responses with PI gains obtained from the Tyreus-Luyblen (TL) tuning rules are illustrated in Fig. 9. Percent overshoot, rise time and settling time are shown in Tab. 5.

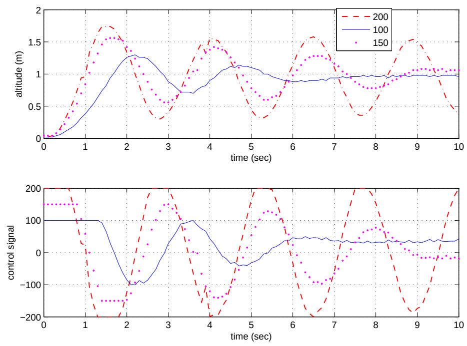

Figure 9: Altitude comparative graphs for PI controller using the TL gains: $K_p = 663$ and $K_i = 25$. Altitude reference = 1,0 m.

The rise time for saturation values of 150 and 200 was too close for both methods, so that

Table 5: Performance parameters measured using three values of saturation. PI tuning gains: $K_p = 663$ $K_i = 25$.

Saturation	$P.O(\%)$	t_r(sec)	t_s(sec)
100	30	1.1	7.4
150	56	0.7	>10
200	74	0.5	>10

the choice of 150 or 200 did not cause significant changes in the performance at low altitudes (<2 m). Another observation is that altitude presented an oscillatory response with high overshoot for both methods. In particular the PI controller tuned using TL rules limited for saturation in 100 presented the best transient behaviour considering the performance parameters evaluated.

It is emphasized that the ZN and TL methods give a previous tuning, so the adjustments for the gains may be required for the altitude response meets the design specifications. Figure 10 shows the altitude response using a P and PI controller with changes in the gains obtained using the TL method. Overshoot, rise time and settling time are shown in Tab. 6. The saturation was 200 for both controllers.

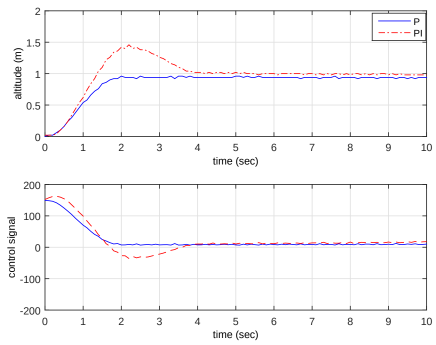

Figure 10: Altitude comparative graphs between heuristic P and PI controllers. P tuning: $K_p = 300$. PI tuning: $K_p = 300$, $K_i = 10$. Altitude reference = 1,0 m.

Table 6: Performance parameters measured for saturation = 200. P tuning: $K_p = 300$. PI tuning: $K_p = 300$, $K_i = 10$.

Controller	$P.O(\%)$	t_r(sec)	t_s(sec)
P	0	1.3	2.0
PI	46	0.9	3.7

It is observed that the altitude response obtained with the P controller did not present overshoot, the settling time was below 3 seconds, and the steady state error was less than 3%. Despite the best performance in relation to the PI controller, the P controller is more sensitive to the battery charge. So that for lower charge there is an increase in the steady state error, which does not occur with the PI controller due to the integral action.

Also the battery charge changes the controller performance. As the battery charge decreases, the overshoot for PI controller decreases and the rise time and settling time increase for both controllers, considering that the PWM reference and the saturation value were not changed.

The altitude response for a trapezoidal reference input is illustrated in Fig. 11. This reference input can be used to control the quadrotor takeoff and landing speed to a certain altitude setpoint. The ramp slope is 10cm/s.

The steady state error and the rise time are lower for the PI controller using ZN method (PI-ZN) with saturation at 100. However, the control effort is greater, as shown in the graph of the control signal, so the energy consumption is higher to maintain the performance. Also for the P controller tuning given in Tab. 6 the control effort is smaller. On the other hand, the tracking capacity of the desired trajectory is reduced. Using the PI controller tuning given in Tab. 6 a smoother behaviour is obtained in the rising stage with less control effort compared to the PI-ZN and PI-TL controllers.

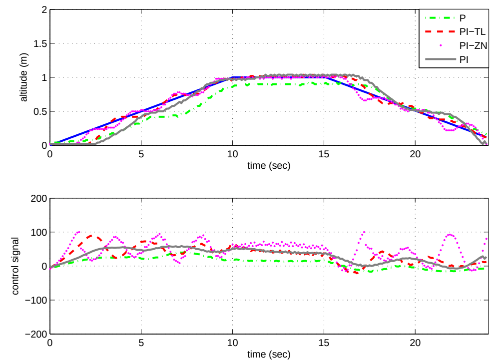

Figure 11: Altitude trajectory comparative.

5 Conclusions

In this paper, we presented an application of the Åström-Hägglund relay method for auto-tuning discrete PID gains of a quadrotor altitude control system.

This proposal was implemented in a real low-cost UAV system and the experimental results showed the potential of the methodology for UAV control. Differently from majority of previous works, our proposal dispenses a mathematical model of the process, providing a simple and efficient solution for UAV altitude control.

Further works may include the replication of the methodology for attitude and position control, and the comparison with other model-based control techniques.

References

Al-Younes, Y. (2009). *Establishing autonomous aus-quadrotor*, Master's thesis, American University of Sharjah.

Alexis, K., Nikolakopoulos, G. and Tzes, A. (2012). Model predictive quadrotor control: attitude, altitude and position experimental studies, *IET Control Theory Applications* **6**(12): 1812–1827.

Åström, K. J. and Hägglund, T. (1984). Automatic tuning of simple regulators, *Proceedings of IFAC 9th World Congress*, pp. 1867–1872.

Bouabdallah, S., Noth, A. and Siegwart, R. (2004). Pid vs lq control techniques applied to an indoor micro quadrotor, *Proceedings of International Conference on Intelligent Robots and Systems*, Vol. 3, pp. 2451–2456.

Dandan, Z., Zhiyun, Z., Meng, Y., Yue, H. and Chen, G. (2014). Application of relay feedback auto-tuning algorithm in a liquid level pid control system, *11th World Congress on Intelligent Control and Automation*, pp. 4313–4315.

Julkananusart, A. and Nilkhamhang, I. (2015). Quadrotor tuning for attitude control based on double-loop pid controller using fictitious reference iterative tuning (FRIT), *Proceedings of 41st Annual Conference of the IEEE Industrial Electronics Society*, pp. 4865–4870.

Kun, D. and Hwang, I. (2016). Linear matrix inequality-based nonlinear adaptive robust control of quadrotor, *Journal of Guidance, Control, and Dynamics* **39**(5): 996–1008.

Lai, L.-C., Yang, C.-C. and Wu, C.-J. (2006). Time-optimal control of a hovering quadrotor helicopter, *Journal of Intelligent and Robotic Systems* **45**(2): 115–135.

Nagaty, A., Saeedi, S., Thibault, C., Seto, M. and Li, H. (2013). Control and navigation framework for quadrotor helicopters, *Journal of Intelligent and Robotic Systems: Theory and Applications* **70**(1-4): 1–12.

Ogata, K. (2010). *Modern Control Engineering*, Instrumentation and controls series, Prentice Hall.

Peng, Y., Vrancic, D. and Hanus, R. (1996). Antiwindup, bumpless, and conditioned transfer techniques for pid controllers, *IEEE Control Systems* **16**(4): 48–57.

Pinto, J. E. M. G. (2014). *Aplicação pratica do metodo de sintonia de controladores pid utilizando o metodo do rele com histerese*, Master's thesis, UFRN, BCZM.

Xiong, J.-J. and Zhang, G. (2016). Discrete-time sliding mode control for a quadrotor uav, *Optik* **127**(8): 3718–3722.

Xue, K., Wang, C., Li, Z. and Chen, H. (2016). Online adaptive error compensation svm-based sliding mode control of an unmanned aerial vehicle, *International Journal of Aerospace Engineering* **2016**.

Yang, J., Cai, Z., Lin, Q. and Wang, Y. (2013). Self-tuning pid control design for quadrotor uav based on adaptive pole placement control, *Chinese Automation Congress (CAC), 2013*, pp. 233–237.

Ziegler, J. G. and Nichols, N. B. (1942). Optimum settings for automatic controls, *Transactions of the A.S.M.E* pp. 759–768.