ACTIVE HEAVE COMPENSATOR USING KALMAN FILTER-BASED DISTURBANCE ESTIMATOR

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Abstract— This paper presents a flatness-based control approach for an active heave compensator actuated by a double acting cylinder. For practical applications, is desired to stabilize the system around the equilibrium point when subjected to unknown external disturbances. Knowing the governing equations, the control system is designed combining the flatness property of the system with a Kalman filter-based disturbance estimator. The efficiency of the proposed control scheme is demonstrated through numerical simulations and compared with related works.

Keywords— Heave compensator; Differential flatness theory; Tracking trajectory; Disturbances compensation

Resumo— Este artigo apresenta uma abordagem de controle baseada na teoria de planicidade diferencial para um compensador de heave atuado por um cilindro de dupla via. Para aplicações práticas, deseja-se estabilizar o sistema em torno do ponto de equilíbro quando sujeito a perturbações externas desconhecidas. A partir das equações governantes, o sistema de controle é projetado combinando a propriedade plana do sistema com um estimador de perturbações baseados no filtro de Kalman. A eficiência do controlador é demonstrada através de simulações numéricas e comparada com trabalhos relacionados.

Palavras-chave— Compensador de heave; Teoria de planicidade diferencial; Acompanhamento de trajetória; Compensação de perturbações.

1 Introduction

In offshore environment, surface vessels are submitted to heave movement (vertical displacements) due to sea waves. In order to reduce the displacement transferred from the vessel to the equipments, a heave compensator is commonly used.

In drilling industries, for instance, the heave displacement of the drill string can generate load variations on the drill bit, swabbing, and buckling of the drill string. Consequently, these problems can collapse the borehole wall, generate well kick and damage the drill bit and the drill string. When the compensator is not able to compensate the heave, the drilling process is halted to avoid the risk of accidents. Stopping the drilling process is equivalent to loss of operation time and money.

Many heave compensators have been proposed in literature. (Hatleskog and Dunnigan, 2007) proposes an active heave compensation system used in conjunction with a passive compensator. The objective is to reduce the heave disturbance at the suspended load above sea floor. In this case, the compensator system behavior is described by a second order differential equation.

In another work, (Do and Pan, 2008) outlines an active heave compensator with a double acting cylinder. The cylinder is operated in such a way that it applies force to the crown block to overcome frictions and the force acting between the riser and the active heave compensation unit to maintain a constant distance from the upper end of the riser to the seabed.

More recently, (Woodacre et al., 2015) provides a review of vertical heave motion compensation systems used on ocean vessels and suggests that more experimental work should be carried out on real-world systems to experimentally validate the active heave compensation controllers being designed and simulated in literature.

In this paper, a flatness-based control approach for an active heave compensator actuated by a double acting cylinder is proposed. The governing equation is a second order differential equation, which describes the dynamic of the suspended mass. The final model is equivalent to a base excited system. The force transmitted to the sprung mass by the vessel's movement is interpreted as a external disturbance, which is estimated by using the Kalman filter recursion as disturbance observer, in addition to state estimation.

This paper is organized as follows. Section 2 presents the mathematical system modeling. Section 3 presents concepts of differential flatness theory and describes the propose control strategy. Numerical simulations results using Matlab/Simulik are presented in Section 4. Lastly, are drawn conclusions and suggestions for further research in Section 5.

2 System Modeling

In this section, a system model similar to the one presented in (Hatleskog and Dunnigan, 2007) is described.

Figure 1 shows the schematic illustration of the system, where X_1 is the sprung mass coordinate, x_0 is the vessel position, k is the stiffness coefficient and c the damping coefficient between the vessel and the sprung mass m. Therefore this is a one degree of freedom system.

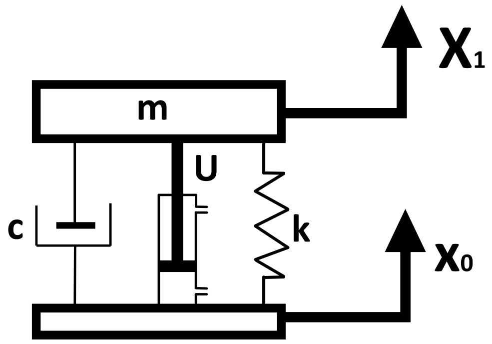

Figure 1: Schematic illustration of the controlled system and the actuator.

The actuator's dynamics, such as saturation and coulomb friction, are not considered in system modeling. It means that, the actuator is able to provide the exact control command required. Moreover, the vessel dynamic in relation to the ocean is not modelled because the active heave compensation system should work for any disturbance, i.e., the active heave compensation system ought to be independent of the vessel movement from ocean waves.

The displacement x_1 can be measured via an inertial measurement unit (IMU), like accelerometers, gyroscopes, magnetometers and combinations of these instruments.

As explained by (Kuchler et al., 2011), between the measurement and the control input there is a delay, which is neglected in this paper. Equation (1) presents the forces acting in the sprung mass m.

$$m\ddot{x}_1 = -kx_1 - c\dot{x}_1 + kx_0 + c\dot{x}_0 + U, \quad (1)$$

where $\dot{x}_1$ is the sprung mass velocity, $\ddot{x}_1$ is the sprung mass acceleration, $\dot{x}_0$ is the vessel velocity and U is the control command.

Equation (1) then can be rewritten into a state-space representation using the states variables x_1 and x_2, as shown in (2).

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\dfrac{kx_1}{m} - \dfrac{c\dot{x}_1}{m} + \dfrac{U}{m} + \dfrac{F_p}{m}, \end{cases} \quad (2)$$

where F_p corresponds an additive disturbance force due to the vessel displacement, and is given by:

$$F_p = kx_0 + c\dot{x}_0. \quad (3)$$

Basically, the control goal is to formulate a state-feedback control law in order to mitigate and compensate disturbances effects. For this purpose, next section presents the control strategy used.

3 Trajectory Tracking Control Design

Commonly found in mobile robotics applications, the flatness property was proposed and developed by (Fliess et al., 1992). A system $\dot{x} = F(x, u)$ with states $x \in \mathbb{R}^n$ and inputs $u \in \mathbb{R}^m$, with $n > m$, is considered differentially flat if is possible to find outputs $z \in \mathbb{R}^m$ as below:

$$z = \xi_z(x, u, \dot{u}, \cdots, u^{(q)}), \quad (4)$$

where z represents the flat outputs of the system and q is a finite number.

Thus, the state and input expressions are represented as follows:

$$x = \xi_x(z, \dot{z}, \cdots, z^{(q)}), \quad (5)$$

$$u = \xi_u(z, \dot{z}, \cdots, z^{(q+1)}), \quad (6)$$

where ξ_z, ξ_x and ξ_u are smooth vector functions and all the components are differentially independents (Fliess et al., 1995).

In order words, a differentially flat system is the one whose system variables can be expressed as a function of these flat outputs and its time derivatives, without the need to solve the integration operation. Moreover, all dynamical systems which satisfy differential flatness properties can be transformed through a change of variables (diffeomorphism) into controllable linear system on Brunovsky's canonical form, whose the flat outputs constitute the state vector (Martin, 2002).

Since creates a local bijection between system state solution and arbitrary trajectories in the flat output space, this differential parametrization plays a key role both in motion planning and in trajectory tracking problems (Sira-Ramírez and Agrawal, 2004). For example, knowing the nominal trajectory $z^*(t)$, one can compute $x^*(t)$ and $u^*(t)$ without solving differential equations by means of (5) and (6), respectively. Thus, flatness allows, in an off-line manner, to check and adjust the desired behavior of system variables with relative ease.

In the followings subsections, first, we exploit the flatness property of the disturbance-free model of (2) to design a feedback control law during trajectory tracking. Thus, assuming the disturbance term affecting the Brunovsky's canonical form, we simultaneous estimate the non-measurable elements of the system state vector as well as the additive disturbance terms with the use of a Kalman filter-based disturbance estimator described in (Rigatos, 2015).

3.1 Brunovsky's canonical form

Assuming $F_p = 0$, it can be shown that, by algebraic manipulation, the system's model given in (2) is a differentially flat system, whose flat output is given by x_1:

$$\begin{cases} x_2 & = \dot{x}_1 \\ U & = m\ddot{x}_1 + kx_1 + c\dot{x}_1. \end{cases} \tag{7}$$

By defining the following new control input

$$\upsilon = -\frac{kx_1}{m} - \frac{c\dot{x}_1}{m} + \frac{U}{m}, \tag{8}$$

the system (2) can be transformed into a linear system on Brunovsky's canonical form, which state-space representation is given by:

$$\dot{\mathbf{X}}_p = \mathbf{A}\mathbf{X}_p + \mathbf{B}\upsilon \ , \ \mathbf{Y} = \mathbf{C}\mathbf{X}_p, \tag{9}$$

with

$$\begin{aligned} \mathbf{X}_p &= \begin{bmatrix} x & \dot{x} \end{bmatrix}^T \quad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ \mathbf{B} &= \begin{bmatrix} 0 & 1 \end{bmatrix}^T \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}. \end{aligned} \tag{10}$$

3.2 Flatness-based controller

Once the system's model is written in the differentially flat form, we define the tracking error $e_1 = x_1 - x_1^*(t)$ for the flat output and its dynamics:

$$\ddot{e}_1 + k_1\dot{e}_1 + k_0 e_1 = 0. \tag{11}$$

As $\ddot{x}_1 = \upsilon$ from (9), we manipulate (11) to propose a suitable state-feedback control law that enables tracking to a desirable trajectory defined by $x_1^*(t)$:

$$\upsilon = \ddot{x}_1^*(t) - k_1\left(\dot{x}_1 - \dot{x}_1^*(t)\right) - k_0\left(x_1 - x_1^*(t)\right), \tag{12}$$

where $k_1, k_0 > 0$ represent the coefficients of a Hurwitz's polynomial $p(s) = s^2 + k_1 s + k_0$ (i.e., a polynomial whose roots are located in the left half-plane of the complex plane). Then assuring an exponentially asymptotically convergence to zero for the tracking error.

3.3 Estimation of Disturbance Forces with Kalman Filtering

The transformation of the system's model into a canonical form through the application of differential flatness theory presented in subsection 3.1, facilitates not only the design of a feedback controller for trajectory tracking but also the design of filters for the estimation of non-measurable elements of the system state vector as well as the estimation of additive disturbance terms affecting the system (Rigatos, 2015).

Next, considering that the additive input disturbance F_p affects the system on Brunovsky's canonical form (i.e. $F_p \neq 0$), we can easily obtain that:

$$\ddot{x}_1 = \upsilon + \delta, \tag{13}$$

where we redefined the disturbance term as $\delta = F_p/m$.

Without loss of generality, the dynamics of the disturbance term is described by their second order derivative and the associated initial conditions. Then, an extended state-space model of equation (13), which comprises as additional state variables the disturbance terms δ and $\dot{\delta}$, can, therefore, be written in matrix form as:

$$\dot{\tilde{\mathbf{X}}}_p = \tilde{\mathbf{A}}\tilde{\mathbf{X}}_p + \tilde{\mathbf{B}}\tilde{\upsilon} \ , \ \mathbf{Y} = \tilde{\mathbf{C}}\tilde{\mathbf{X}}_p, \tag{14}$$

with

$$\begin{aligned} \tilde{\mathbf{X}}_p &= \begin{bmatrix} x \\ \dot{x} \\ \delta \\ \dot{\delta} \end{bmatrix} \quad \tilde{\upsilon} = \begin{bmatrix} \upsilon \\ \ddot{\delta} \end{bmatrix} \quad \tilde{\mathbf{A}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \tilde{\mathbf{B}} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T \quad \tilde{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{15}$$

For the extended state-space description of the system one can design a state estimator of the form

$$\dot{\hat{\tilde{\mathbf{X}}}}_p = \tilde{\mathbf{A}}_o\hat{\tilde{\mathbf{X}}}_p + \tilde{\mathbf{B}}_o\tilde{\upsilon} + \mathbf{K}_o(Y^{meas} - \tilde{\mathbf{C}}_o\hat{\tilde{\mathbf{X}}}_p), \tag{16}$$

where for matrices $\tilde{\mathbf{A}}_o$ and $\tilde{\mathbf{C}}_o$ holds $\tilde{\mathbf{A}}_o = \tilde{\mathbf{A}}$ and $\tilde{\mathbf{C}}_o = \tilde{\mathbf{C}}$, while for matrix $\tilde{\mathbf{B}}_o$ one has

$$\tilde{\mathbf{B}}_o = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T. \tag{17}$$

In addition, the estimator's gain $K_o \in \mathbb{R}^{4\times 1}$ will be computed through the Kalman Filter recursion. Before that, we define $\tilde{\mathbf{A}}_d$, $\tilde{\mathbf{B}}_d$ and $\tilde{\mathbf{C}}_d$ as the discrete-time equivalents of matrices $\tilde{\mathbf{A}}_o$, $\tilde{\mathbf{B}}_o$ and $\tilde{\mathbf{C}}_o$, respectively. Then, the associated Kalman filter-based disturbance estimator is given by (Rigatos, 2015):

measurement update:

$$\begin{aligned} \mathbf{K}(k) &= \mathbf{P}^-(k)\tilde{\mathbf{C}}_d^T\left[\tilde{\mathbf{C}}_d\mathbf{P}^-(k)\tilde{\mathbf{C}}_d^T + \mathbf{R}\right]^{-1} \\ \hat{\tilde{\mathbf{X}}}_p(k) &= \hat{\tilde{\mathbf{X}}}_p^-(k) + \mathbf{K}(k)\left[Y^{meas} - \tilde{\mathbf{C}}_d\hat{\tilde{\mathbf{X}}}_p\right] \\ \mathbf{P}(k) &= \mathbf{P}^-(k) - \mathbf{K}(k)\tilde{\mathbf{C}}_d\mathbf{P}^-(k) \end{aligned} \tag{18}$$

time update:

$$\begin{aligned}\mathbf{P}^{-}(k+1) &= \tilde{\mathbf{A}}_d\mathbf{P}(k)\tilde{\mathbf{A}}_d^T + \tilde{\mathbf{B}}_d\mathbf{Q}\tilde{\mathbf{B}}_d^T \\ \tilde{\mathbf{X}}_p^{-}(k+1) &= \tilde{\mathbf{A}}_d\tilde{\mathbf{X}}_p(k) + \tilde{\mathbf{B}}_d\tilde{\upsilon}(k).\end{aligned} \tag{19}$$

Then, to compensate for the effects of the disturbance force it suffices to modify the control input (12) that is applied to (9) by:

$$\upsilon = \ddot{x}_1^*(t) - k_1\left(\dot{x}_1 - \dot{x}_1^*(t)\right) - k_0\left(x_1 - x_1^*(t)\right) - \tilde{\delta}, \tag{20}$$

where $\tilde{\delta}$ corresponds to the estimated disturbance term through the Kalman Filter recursion.

Finally, as $\ddot{x}_1 = \upsilon$ from (9), the control law υ is transformed into the original control input U as follows:

$$U = m\upsilon + kx_1 + c\dot{x}_1. \tag{21}$$

4 Numerical Simulations

This section presents the simulation results obtained from the propose control strategy. The main goal is stabilize the system at constant trajectory equal to zero (i.e. the equilibrium point). Table 1 shows the system's parameters used in the simulations, which were taken and adapted from (Cuellar and Fortaleza, 2015).

Table 1: System's parameters

Symbol	Definition	Value
m	mass	286 *tonnes*
K	stiffness	60.2 kN/m
ζ	damping ratio	0.1
c	damping coefficient	13.12 kNs/m
T_s	sample time	0.0737 s

The parameters for the Kalman filter were set as follows:

$$\mathbf{R} = 0.0001, \quad \mathbf{Q} = 1, \tag{22}$$

where $\mathbf{R}$ is the measurement noise covariance matrix and $\mathbf{Q}$ is the process noise covariance matrix. Therefore, we assume that the measured variable x_1 is affected by an additive white Gaussian noise with variance equal to 0.0001 (standard deviation $\sigma = 0.01\ m$).

The controller gains were chosen as the coefficients of a Hurwitz's polynomial characterized by having two roots equal to $-1.5 \pm 1.5i$. These gains provide a reasonable attenuation rate as also an acceptable control signal.

The first simulation results show how the controlled system responds to a periodic disturbance of low frequency. The disturbance used is a composition of two harmonic functions:

$$\begin{aligned}\delta(t) &= 0.35sin(0.35t) + 0.25cos(0.5t) \\ &+ 0.25sin(0.5t) + 0.35cos(0.75t).\end{aligned} \tag{23}$$

Figure 2 presents the vessel's disturbance (blue line) and the sprung mass response (red line). The upper graphic of Figure 3 presents the signal control requirement to overcome the disturbance. In the lower graphic is shown the F_p signal (blue line) and the estimated F_p signal (red line). Remembering that F_p is δ multiplied by m. Figure 4 presents the system's states x_1 and x_2. The upper graphic shows the desired constant trajectory (equal to zero) for state x_1 (blue line), and the state x_1 response under the disturbance (red line). The lower graphic shows the same for state x_2.

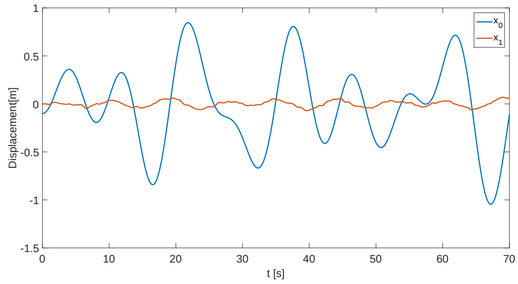

Figure 2: Periodic disturbance and the response of the controlled system

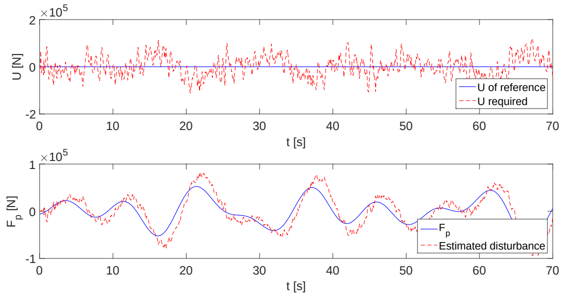

Figure 3: Control signal and F_p

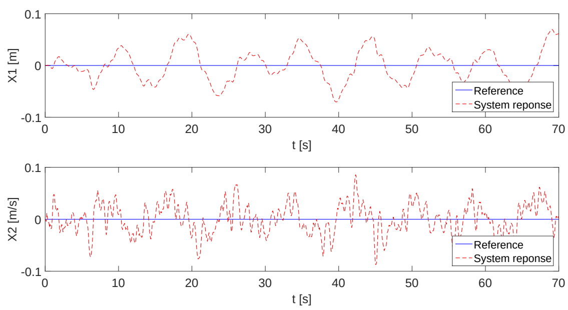

Figure 4: Time evolution of the sprung mass position x_1 and velocity x_2

For quantitative analysis, we calculated the attenuation between the maximum input and the maximum output. Considering this situation as the most critical, the result obtained is 93.25% of attenuation.

The second simulation uses an random disturbance taken from (Ni et al., 2009). Figures 5-7 do the same analysis that was done for Figures 2-4, respectively, but now for a random disturbance, which has in its composition high frequencies.

The attenuation obtained for the random disturbance is 85.4%. It is an acceptable result once (Cuellar and Fortaleza, 2015) obtains 75% and (Ni et al., 2009) obtains 83.17% of attenuation. It is important to be aware that (Cuellar and Fortaleza, 2015) proposes a semi-active control and (Ni et al., 2009) proposes a passive control. (Hatleskog and Dunnigan, 2007) suggests that the maximum attenuation for an active heave compensator is around 95%.

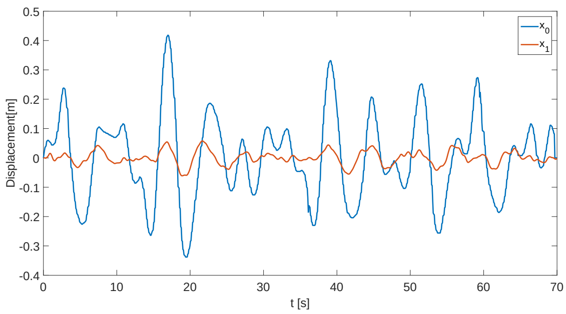

Figure 5: Random disturbance and the response of the controlled system

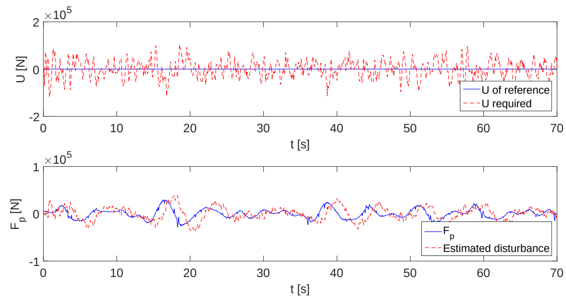

Figure 6: Control signal and F_p

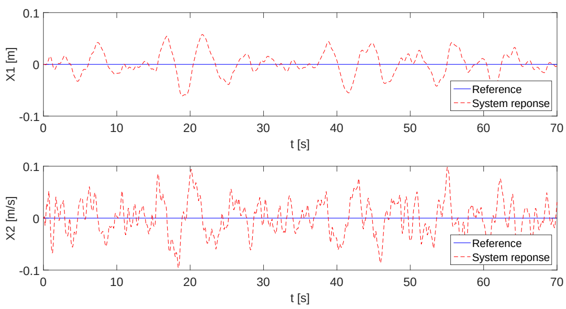

Figure 7: Time evolution of the sprung mass position x_1 and velocity x_2

In both tests, for the control signal, we obtained a relation of 1/2.5 between the control signal and the sprung mass, which can be considered reasonable for practical applications.

5 Conclusions

In this paper, a flatness-based control approach for an active heave compensator actuated by a double acting cylinder was proposed. Estimation of disturbance terms affecting the system has been performed with the use of a Kalman filter.

First, it was proven that the dynamic model of the active heave compensator is a differentially flat one, and this enabled its description in the Brunovsky canonical form. For this equivalent system, it has been shown that it is possible to design a state feedback controller. At a second stage, by redesigning the Kalman Filter in the form of a disturbance observer, it has been also shown that once an estimation of the unknown disturbance inputs is obtained their effect can be compensated by an additional element, which is based on the estimated value of the disturbance forces, that is included in the control loop to improve system robustness.

The performance of the proposed control scheme has been evaluated through numerical simulation tests, providing an reasonable attenuation rates with an acceptable control signal for different kinds of external disturbances. Further studies are needed to include the actuator's dynamics on the global model.

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