EVENT-TRIGGERED CONTROL FOR RATIONAL PLANTS

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Abstract— In this work, we deal with the problem of designing stabilizing event-triggered state-feedback controllers for rational plants. Using differential algebraic representations and Lyapunov theory techniques, we derive LMI-based conditions to ensure regional asymptotic stability of the origin. These conditions can be cast in convex optimization problems to choose the parameters of the trigger function aiming at reducing the sampling activity while ensuring the asymptotic stability of the origin with respect to a given set of admissible initial conditions. The application of the method is shown by means of simulations.

Keywords— Event-triggered Control, Rational Systems, Networked Control Systems.

Resumo— Neste trabalho, trata-se do problema de projetar um sistema de controle por realimentação de estados baseado em eventos (*event-triggered*) para plantas não-lineares racionais. Usando representações algébrico-diferenciais e técnicas da teoria de Lyapunov, condições LMI que garantem a estabilidade da origem são elaboradas. Estas condições são então usadas em um problema de otimização convexo para ajustar os parâmetros do gerador de eventos visando reduzir o número de amostragens e garantir a estabilidade assintótica da origem para um conjunto admissível de condições iniciais dado. A aplicação do método é ilustrada através de simulações.

Palavras-chave— Controle baseado em eventos, Sistemas Racionais, Sistemas de Controle em Rede.

1 Introduction

Event-triggered control is a paradigm of sampled control where the data is sampled and the control is updated only when a criterion based on the state or output of the system is violated. This way, to achieve a given control objective, typically less updates are needed than with a periodic sampling paradigm. In a networked control implementation, this represents less transmissions across the network, reducing the bandwidth and energy consumption (Heemels et al. (2012), Abdelrahim, Postoyan, Daafouz and Nešić (2015a), Hespanha et al. (2007) and the references cited therein).

Event-triggered control strategies have been addressed in the context of linear time invariant plants in (Heemels et al., 2012), which also addresses self-triggered control, and in (Donkers and Heemels, 2012), with an emphasis on decentralized triggering. Event-triggered implementations of PI controllers are addressed in (Gomes da Silva Jr. et al., 2014) for linear plants and in (Moreira et al., 2016) for linear plants subject to saturation of the control input. Stability analysis of event-triggered PI controllers for first-order stable plants is also presented in (Tiberi et al., 2012) and (Beschi et al., 2014).

In the context of nonlinear plants, (Tabuada, 2007) shows a trigger criterion that renders the closed-loop system stable and proves that this criterion results in lower-bounded inter-event times. More recently, (Abdelrahim, Postoyan and Daafouz, 2015) addresses the event-triggered control of singularly perturbed systems when the trigger has access to the entire state of the system, while (Abdelrahim, Postoyan, Daafouz and Nešić, 2015a), (Abdelrahim et al., 2014) and (Abdelrahim, Postoyan, Daafouz and Nešić, 2015b) address event-triggered strategies considering output feedback control laws where the trigger has access only to the system outputs. These papers use hybrid systems theory to prove stability and include a timer to guarantee a minimum inter-event time (dwell-time technique).

In the present paper, we address the design of event-triggered control systems for rational plants. Many plants of interest can be modeled as rational, notably in the biological area, e.g. generic biological systems (Wu and Mu, 2009) and bio-reactors (Campestrini et al. (2014), Antonelli et al. (2003)). We consider a triggering strategy based on a weighted relative distance between the last sampled state and the current continuous one; the weights are introduced through the use of positive definite matrices. Using differential algebraic representations (Trofino (2000), Coutinho et al. (2004), Oliveira (2012)) and the Lyapunov theory, LMI conditions that guarantee the stability of the closed-loop system for a set of admissible initial states are derived. A convex optimization problem is then proposed as means to tune the parameters of the trigger function aiming at reducing the number of events while keeping the closed-loop system stable. We cover the so-called emulation case (Heemels et al., 2012), i.e. when the controller gain matrix is given *a priori* and only the trigger parameters are obtained using the optimization problem. Numerical experiments illustrate the application of the method.

Notation: $\mathbb{R}$ represents the set of real numbers. A' denotes the transpose of matrix A.

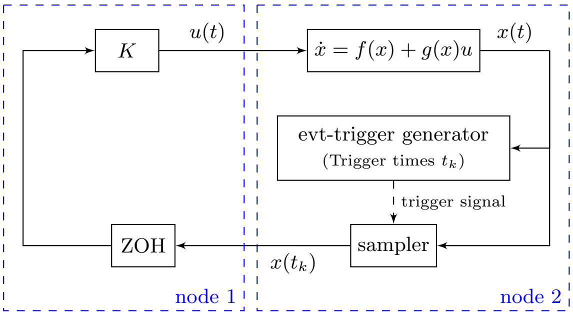

Figure 1: Closed-loop system diagram

The symbol $*$ stands for symmetric blocks within a matrix. $\mathrm{tr}(A)$ denotes the trace of matrix A. $\mathrm{diag}(X, Y)$ denotes the block-diagonal matrix composed by the blocks X and Y. $A_{(i)}$ and $x_{(i)}$ denote the i-th row of matrix A and vector x, respectively. 0 is used to represent matrices of null entries of appropriate dimensions or the scalar zero. $\|\cdot\|$ denotes the Euclidean Norm and $|\cdot|$ denotes the absolute value. $\mathcal{E}(P) = \{\xi \in \mathbb{R}^n : \xi' P \xi \leq 1\}$ denotes an ellipsoid defined from a matrix $P = P' > 0 \in \mathbb{R}^{n \times n}$.

2 Problem Statement

Consider an autonomous continuous-time control system defined by the following equation:

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t) \\ u(t) = Kx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the input; $f(x)$ and $g(x)$ are rational functions of the state and are Lipschitz continuous on the region of interest $\mathcal{B}_x$. We assume that $f(0) = 0$ and that the equilibrium point of interest is the origin. $K \in \mathbb{R}^{m \times n}$ is a constant gain matrix that stabilizes the origin of the system.

We consider now that the plant and the controller are in separate nodes and are connected through a general purpose network forming the closed-loop system depicted in Figure 1. At instants t_k, $k = 0, 1, 2, \ldots$, determined by the event-trigger generator, a sample of the plant state is sent to the controller node. Between two trigger instants, the controller input is held at the constant sampled value by means of a zero-order holder. We assume $t_0 = 0$. Therefore, the closed loop system can be represented by the equation:

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t) \\ u(t) = Kx(t_k) \qquad \forall t \in [t_k, t_{k+1}) \end{cases} \tag{2}$$

We aim at designing an event-trigger strategy that saves communication bandwidth by transmitting data only when necessary, while keeping the closed-loop system stable. In a more formal way we can state the following problem to be addressed:

Problem 1 *Given a static state-feedback matrix K that stabilizes the origin of the closed-loop system (1), (i.e., a continuous-time implementation of the control system), devise an event-triggered strategy that guarantees the asymptotic stability of the origin for the corresponding event-triggered implementation (2) for all initial conditions in a given subset $\mathcal{X}_0$ of the domain of attraction of the origin, while reducing the number of trigger events.*

3 Preliminaries

Defining $\delta(t) = x(t_k) - x(t)$, we can rewrite (2) as follows:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))K(x(t) + \delta(t)) \\ &= f(x(t)) + g(x(t))Kx(t) + g(x(t))K\delta(t) \\ &\qquad\qquad \forall t \in [t_k, t_{k+1}) \end{aligned} \tag{3}$$

Note that at $t = t_k$, the value of $\delta(t)$ is reset to zero.

In this paper we will use the differential algebraic representations (DARs) proposed in (Trofino, 2000). A DAR for (3) is given by:

$$\begin{cases} \dot{x} = (A_1(x) + A_3(x)K)\,x + A_2(x)\xi(x) + A_3(x)K\delta \\ 0 = (\Omega_1(x) + \Omega_3(x)K)\,x + \Omega_2(x)\xi(x) + \Omega_3(x)K\delta \end{cases} \tag{4}$$

with $\xi(x) \in \mathbb{R}^q$ being an auxiliary variable containing nonlinear terms of $f(x)$ and of $g(x)$; $A_1(x) \in \mathbb{R}^{n \times n}$, $A_2(x) \in \mathbb{R}^{n \times q}$, $A_3(x) \in \mathbb{R}^{n \times m}$, $\Omega_1(x) \in \mathbb{R}^{q \times n}$, $\Omega_2(x) \in \mathbb{R}^{q \times q}$ and $\Omega_3(x) \in \mathbb{R}^{q \times m}$ being affine matrix functions of x. It is assumed that (3) can be recovered from (4) by eliminating $\xi(x)$, which implies that $\Omega_2(x)$ is full column rank for all x of interest, i.e. $\forall x \in \mathcal{B}_x$. We omitted the time dependency to simplify notation.

Notice that, since $f(x)$ and $g(x)$ are rational functions of the state, so is $f(x) + g(x)Kx$, and the decomposition of (3) in the form (4) is always possible. The decomposition is not unique and different choices can lead to different results (Coutinho et al. (2004), Trofino (2000)).

4 Sampling strategy

We consider the triggering strategy given by the following algorithm (Moreira et al. (2016), Groff et al. (2016)):

Algorithm 1 Event-trigger strategy

if $\delta'(t)Q_\delta\delta(t) - x'(t)Q_x x(t) > 0$ **then**
  $t_k = t$;
  $u(t) = Kx(t_k)$;
else
  $u(t) = u(t_k)$;
end if

The term $\delta'(t)Q_\delta\delta(t) - x'(t)Q_x x(t)$ is a relative measure of the deviation between the last sampled state and the current one. Q_δ and Q_x are symmetric positive definite matrices of appropriate dimensions that act as weights on this measure. The "larger" Q_x and the "smaller" Q_δ are, the more we let the current state deviate from the last sampled one before sampling again and less sampling activity is expected. This criterion is a generalization of the sampling criterion originally introduced in (Tabuada, 2007). Notice that if we choose $Q_x = \sigma I$ and $Q_\delta = \mu I$ we basically retrieve the criterion in (Tabuada, 2007) with $\sigma_0 = \sigma/\mu$ and, as shown in that paper, the larger σ_0 is, less sampling activity is expected.

5 Stability conditions

Assuming that the gain K has been designed to stabilize the closed-loop system (1) (i.e., a continuous-time implementation of the control system), the following theorem establishes conditions to ensure the stability of the event-triggered implementation using the sampling strategy given by Algorithm 1:

Theorem 1 *Consider a convex polytopic region of interest $\mathcal{B}_x = \{x \in \mathbb{R}^n : h_i'x \leq \rho_i;$ with $\rho_i > 0;$ $h_i \in \mathbb{R}^n;$ $i = 1, \ldots, n_f;\}$, whose vertices are $\{x_j \in \mathbb{R}^n;\ j = 1, \ldots, n_v\}$. If there exist constant symmetric positive definite matrices $P, \bar{Q}_x, Q_\delta \in \mathbb{R}^{n\times n}$ and constant matrices $M_1, M_2 \in \mathbb{R}^{n\times n}$ and $M_3 \in \mathbb{R}^{q\times q}$, such that the following LMIs are satisfied:*

$$\begin{bmatrix} \psi_1 & \psi_2 & \psi_3 & \psi_4 & I \\ * & \psi_5 & \psi_6 & \psi_7 & 0 \\ * & * & \psi_8 & \psi_9 & 0 \\ * & * & * & -Q_\delta & 0 \\ * & * & * & * & -\bar{Q}_x \end{bmatrix} < 0, \quad j = 1 \ldots n_v \tag{5}$$

$$\begin{bmatrix} P & h_i \\ h_i' & \rho_i^2 \end{bmatrix} > 0, \quad i = 1 \ldots n_f \tag{6}$$

where

$$\psi_1 = \mathrm{He}\{M_2(A_1(x_j) + A_3(x_j)K)\}$$
$$\psi_2 = P + (A_1(x_j) + A_3(x_j)K)'M_1' - M_2$$
$$\psi_3 = M_2A_2 + (\Omega_1(x_j) + \Omega_3(x_j)K)'M_3'$$
$$\psi_4 = M_2A_3(x_j)K \qquad \psi_5 = -\mathrm{He}\{M_1\}$$
$$\psi_6 = M_1A_2(x_j) \qquad \psi_7 = M_1A_3(x_j)K$$
$$\psi_8 = \mathrm{He}\{M_3\Omega_2(x_j)\} \qquad \psi_9 = M_3\Omega_3(x_j)K$$

then $\forall x(0) \in \mathcal{E}(P) = \{x \in \mathbb{R}^n : x'Px \leq 1\}$, the trajectory of system (2) under the sampling strategy given by Algorithm 1 with $Q_x = \bar{Q}_x^{-1}$ converges asymptotically to the origin, i.e. $\mathcal{E}(P)$ is included in the region of attraction of the origin.

Proof: This proof uses a procedure similar to that presented in (Oliveira, 2012, Chapter 6).

We can rearrange the terms of the representation (4) as follows:

$$\begin{cases} -\dot{x} + (A_1 + A_3K)x + A_2\xi + A_3K\delta = 0 \\ (\Omega_1 + \Omega_3K)x + \Omega_2\xi + \Omega_3K\delta = 0 \end{cases} \tag{7}$$

where we omitted the matrix dependencies on x for notation clarity. From this, along the solutions of (4), it follows that:

$$\beta_1 = \dot{x}'M_1(-\dot{x} + (A_1 + A_3K)x + A_2\xi + A_3K\delta) = 0$$
$$\beta_2 = x'M_2(-\dot{x} + (A_1 + A_3K)x + A_2\xi + A_3K\delta) = 0$$
$$\beta_3 = \xi'M_3((\Omega_1 + \Omega_3K)x + \Omega_2\xi + \Omega_3K\delta) = 0$$

$\forall M_1, M_2, M_3$ of appropriate dimensions.

Consider now a quadratic Lyapunov function $V(x) = x'Px$, with P symmetric positive definite. Defining $\zeta = [x'\ \dot{x}'\ \xi'\ \delta']'$ and considering β_1, β_2 and β_3 above, it follows that $\dot{V}(x)$ can be written as:

$$\dot{V}(x) = \dot{V}(x) + 2\beta_1 + 2\beta_2 + 2\beta_3 = \\ = \zeta' \begin{bmatrix} \psi_a & \psi_b & \psi_c & M_2A_3K \\ * & -\mathrm{He}\{M_1\} & M_1A_2 & M_1A_3K \\ * & * & \mathrm{He}\{M_3\Omega_2\} & M_3\Omega_3K \\ * & * & * & 0 \end{bmatrix} \zeta \tag{8}$$

with $\psi_a = \mathrm{He}\{M_2(A_1 + A_3K)\}$, $\psi_b = P + (A_1 + A_3K)'M_1' - M_2$ and $\psi_c = M_2A_2 + (\Omega_1 + \Omega_3K)'M_3'$.

From Algorithm 1, it follows that, along the trajectories of the system, $\delta'Q_\delta\delta - x'Q_xx \leq 0$. Then, if the following matrix inequality is verified:

$$\begin{bmatrix} \psi_a + Q_x & \psi_b & \psi_c & M_2A_3K \\ * & -\mathrm{He}\{M_1\} & M_1A_2 & M_1A_3K \\ * & * & \mathrm{He}\{M_3\Omega_2\} & M_3\Omega_3K \\ * & * & * & -Q_\delta \end{bmatrix} < 0 \tag{9}$$

we can conclude that $\dot{V}(x) < \delta'Q_\delta\delta - x'Q_xx \leq 0$.

Applying the Schur's complement to $\psi_a + Q_x$, we obtain that (5) and (9) are equivalent. Hence, satisfaction of (5) implies that $\dot{V}(x) < 0$. Since (5) is affine on x, satisfying it on the vertices of $\mathcal{B}_x$ implies that it is satisfied in the entire $\mathcal{B}_x$ and that the origin is asymptotically stable.

Satisfaction of (6) guarantees that $\mathcal{E}(P) \subset \mathcal{B}_x$ (see Tarbouriech et al. (2011) for details), making $\mathcal{E}(P)$ an invariant and contractive set for the nonlinear system (2) and guaranteeing that $\forall x(0) \in \mathcal{E}(P)$ converges asymptotically to the origin. $\square$

6 Zeno Solutions

To be of practical use, the triggering schema cannot lead to Zeno solutions (Abdelrahim, Postoyan, Daafouz and Nešić (2015a), Tabuada (2007), Heemels et al. (2012)). The following theorem addresses this issue.

Theorem 2 *The inter-sampling times implicitly defined by the triggering schema given by Algorithm 1 are lower-bounded.*

Proof: Observe that the trigger criterion can be rewritten as:

$$\text{Trigger if } \frac{\delta' Q_\delta \delta}{x' Q_x x} > 1$$

When a trigger event occurs, δ becomes 0 and a new trigger event will occur only when the relation above is satisfied again. That is, no new event can occur while $\frac{\delta' Q_\delta \delta}{x' Q_x x} \leq 1$. Hence, noticing that $\frac{\delta' Q_\delta \delta}{x' Q_x x} \leq \lambda \frac{\|\delta\|^2}{\|x\|^2}$, with $\lambda = \frac{\lambda_{max}(Q_\delta)}{\lambda_{min}(Q_x)}$; no new trigger can occur while:

$$\lambda \left(\frac{\|\delta\|}{\|x\|} \right)^2 < 1 \iff \frac{\|\delta\|}{\|x\|} < \sqrt{\frac{1}{\lambda}}$$

From this point, the proof follows the steps presented in (Tabuada, 2007, Theorem III.1). Consider the dynamics of $\frac{\|\delta\|}{\|x\|}$ between two sampling instants:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\|\delta\|}{\|x\|}\right) &= -\frac{\delta'\dot{x}}{\|\delta\|\|x\|} - \frac{x'\dot{x}}{\|x\|^2}\frac{\|\delta\|}{\|x\|} \leq \\ &\leq \frac{\|\delta\|\|\dot{x}\|}{\|\delta\|\|x\|} + \frac{\|x\|\|\dot{x}\|}{\|x\|^2}\frac{\|\delta\|}{\|x\|} = \left(1 + \frac{\|\delta\|}{\|x\|}\right)\frac{\|\dot{x}\|}{\|x\|} = \\ &= \left(1 + \frac{\|\delta\|}{\|x\|}\right)\frac{\|h(x, x+\delta)\|}{\|x\|} \leq \\ &\leq \left(1 + \frac{\|\delta\|}{\|x\|}\right)\frac{L\|x\| + L\|\delta\|}{\|x\|} = L\left(1 + \frac{\|\delta\|}{\|x\|}\right)^2 \end{aligned} \tag{10}$$

where $h(x, x+\delta) \triangleq f(x) + g(x)K(x+\delta)$ and L is the Lipschitz constant of h in the set $\{[x' \ \delta']' \in \mathbb{R}^{2n} : x \in \mathcal{B}_x, \ \delta' Q_\delta \delta - x' Q_x x \leq 0\}$. The last inequality in (10) comes from the fact that since $f(x)$ and $g(x)$ are Lipschitz, so is $h(x, x+\delta)$, thus:

$$\begin{aligned} &\|h(r_1, r_1 + s_1) - h(r_2, r_2 + s_2)\| \leq \\ &\leq L\|(r_1, s_1) - (r_2, s_2)\| \end{aligned} \tag{11}$$

and, taking $r_1 = x$, $s_1 = \delta$, $r_2 = s_2 = 0$:

$$\begin{aligned} &\|h(x, x+\delta) - h(0,0)\| = \\ &= \|h(x, x+\delta)\| \leq \\ &\leq L\|(x,\delta) - (0,0)\| = L\|(x,\delta)\| \leq \\ &\leq L\|x\| + L\|\delta\| \end{aligned} \tag{12}$$

Defining $\varphi \triangleq \frac{\|\delta\|}{\|x\|}$, (10) becomes:

$$\dot{\varphi} \leq L(1+\varphi)^2 \tag{13}$$

Thus, $\frac{\delta' Q_\delta \delta}{x' Q_x x}$ takes more time to go from 0 to 1 than $\varphi(t)$, solution of the initial value problem $\dot{\varphi} = L(1+\varphi)^2$ with $\varphi(0) = 0$, takes to reach $\sqrt{1/\lambda}$ for the first time. The solution to this initial value problem is given by:

$$\varphi(t) = \frac{Lt}{1 - Lt} \tag{14}$$

which is continuous at $t = 0$. Therefore, it takes a finite (as opposed to infinitesimal) amount of time to the value $\sqrt{1/\lambda}$ be reached. Thus, we conclude that the inter-sampling times are lower-bounded by this amount. □

Hence, from Theorem 2, we can guarantee that the proposed triggering strategy does not generate Zeno solutions.

7 Trigger tuning

Consider a given region of initial states $\mathcal{X}_0 = \{x \in \mathbb{R}^n : x' P_0 x \leq 1\}$, with $P_0 \in \mathbb{R}^{n \times n}$ symmetric and positive definite, from where we want to ensure convergence of the trajectories to the origin. Satisfaction of conditions stated in Theorem 1 and $\mathcal{X}_0 \subset \mathcal{E}(P)$ are sufficient conditions to achieve this goal.

To reduce the sampling activity, we aim at finding Q_δ as "small" as possible and Q_x as "large" as possible. The goal on Q_δ can be accomplished, for example, by minimizing $\text{tr}(Q_\delta)$. Since $\bar{Q}_x = Q_x^{-1}$ appears in condition (5), the goal on Q_x can be indirectly achieved by minimizing $\text{tr}(\bar{Q}_x)$. Thus, considering both objectives, we propose the following optimization problem as means of computing the trigger parameters:

$$\begin{aligned} &\min_{\{P, \bar{Q}_x, Q_\delta, M_1, M_2, M_3\}} (\text{tr}(Q_\delta + \bar{Q}_x)) \\ &\text{subject to:} \qquad (5),\ (6),\ P < P_0 \end{aligned} \tag{15}$$

The last constraint in (15) ensures that $\mathcal{X}_0 \subset \mathcal{E}(P)$.

Notice that the choice of $\mathcal{B}_x$ impacts on the results obtained with this optimization problem. For this reason, one should parameterize $\mathcal{B}_x$ (e.g. as a symmetrical polytope around the origin) and do a linear search on the parameter (see the numerical example).

8 Numerical example

Consider the following plant:

$$\begin{cases} \dot{x}_a(t) = x_b(t) \\ \dot{x}_b(t) = \left(1 + x_a^2(t)\right) x_a(t) + x_b(t) + u(t) \end{cases} \tag{16}$$

where $x = [x_a \ x_b]' \in \mathbb{R}^2$ is the plant state. There is only one equilibrium point, the origin, and it is a saddle point.

A local stabilizing controller gain matrix is given by $K = \begin{bmatrix} -2 & -2 \end{bmatrix}$. In this case, one possible DAR of the form (4), is given by $\xi(x) = x_a^2$ and the following matrices:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 \\ x_a \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\Omega_1 = \begin{bmatrix} x_a & 0 \end{bmatrix} \quad \Omega_2 = \begin{bmatrix} -1 \end{bmatrix} \quad \Omega_3 = \begin{bmatrix} 0 \end{bmatrix}$$

Considering $\mathcal{B}_x = \{x \in \mathbb{R}^2 : |x_{(i)}| < \rho\}$ and $\mathcal{X}_0 = \{x \in \mathbb{R}^2 : x' P_0 x \leq 1\}$, with $P_0 = 50I$, and applying the optimization problem (15) for various values of parameter ρ (i.e., various sizes of $\mathcal{B}_x$), we obtain the results shown in Table 1.

Table 1: Linear search on $\mathcal{B}_x$ size

	ρ	$\mathrm{tr}(Q_\delta + \bar{Q}_x)$
	0.10	unfeasible
	0.20	111.622
	0.40	38.268
	0.50	30.813
$*$	0.60	28.358
	0.70	29.563
	0.80	36.038
	0.90	unfeasible

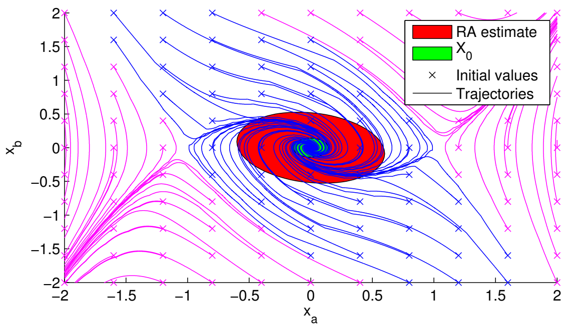

Figure 2: Phase portrait – K given

The asterisk in this table indicates the line corresponding to the best value found for the objective function. With this value of ρ, the optimization problem yields:

$$Q_x = \begin{bmatrix} 0.331751 & -0.118657 \\ -0.118657 & 0.593763 \end{bmatrix}$$

$$Q_\delta = \begin{bmatrix} 11.6490 & 11.6489 \\ 11.6489 & 11.6490 \end{bmatrix}$$

$$P = \begin{bmatrix} 2.86019 & 0.552851 \\ 0.552851 & 3.70859 \end{bmatrix}$$

Using these values for the trigger parameters, simulations of the closed-loop system give the results shown in figures 2 and 3. Figure 2 shows the set $\mathcal{X}_0$ (in green) along with the set $\mathcal{E}(P)$ (in red), some convergent (in blue) and some divergent trajectories (in magenta). Note that the set $\mathcal{X}_0$ is included in the domain of attraction of the origin and in $\mathcal{E}(P)$, as expected. The estimate of the region of attraction given by $\mathcal{E}(P)$ is somewhat conservative, as expected, since the optimization problem does not aim at maximizing it. In Figure 3, the top plot shows the time evolution of a simulation with $x(0) = [0.1\ 0.1]'$ and the bottom plot shows the corresponding trigger activity. In the time interval $[0, 15]$, the trigger generated 64 events with a minimum inter-event time of 144 ms.

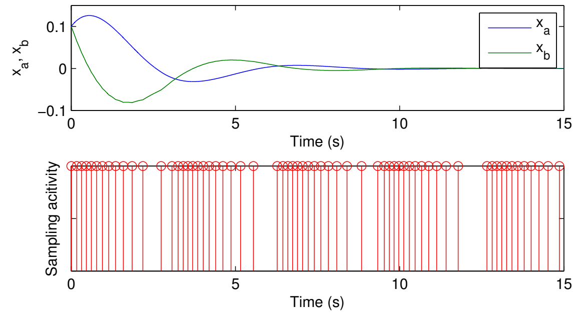

Figure 3: Simulation for $x(0) = [0.1\ 0.1]'$

9 Concluding remarks

In this work we have proposed a method to design event-triggered control systems for continuous-time rational nonlinear plants. Differently from previous works, we do not rely on hybrid systems theory to prove stability and we do not use dwell-time techniques to ensure a minimum inter-event time. The method is based on the use of differential algebraic representations for rational systems and guarantees local asymptotic stability of the origin for a given set of initial conditions. The stability conditions derived are LMIs. Therefore, a convex optimization problem is proposed to compute the trigger parameters aiming at reducing the number of trigger events. We addressed the so-called emulation design, where the controller gains are given and the design task is to tune only the trigger function parameters. A numerical example showed the potentialities of the method.

The co-design of the trigger parameters and the controller gains and use of less conservative design approaches are subjects of ongoing work.

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