JUMP LINEAR QUADRATIC OPTIMAL CONTROL APPLIED TO A WIND TURBINE GENERATOR SYSTEM

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Abstract— This paper deals with the optimal control problem of a wind turbine generator system (WTGS). A Markov jump linear system (MJLS) model is built for the WTGS from several interconnected subsystems: a rotor, a pitch actuator, a mechanical, and an electrical subsystem. A linear quadratic optimal control problem is proposed and results of a numerical simulation are presented.

Keywords— Wind Turbine Generator Systems, Optimal Filtering, Optimal Control, Markov Jump Linear Systems, General State Space, Riccati Equations.

Resumo— Este artigo trata do problema de controle ótimo de um sistema de geração eólica (WTGS). O aerogerador é descrito como um sistema linear com saltos markovianos (MJLS) e seu modelo é obtido a partir de vários subsistemas interligados: rotor, atuador de ângulo de passo das pás, mecânico e elétrico. Um problema de controle ótimo linear-quadrático é proposto e os resultados de uma simulação numérica são apresentados.

Palavras-chave— Aerogeradores, Filtragem Ótima, Controle Ótimo, Sistemas Lineares com Saltos Markovianos, Espaço de Estados Geral, Equações de Riccati.

1 Introduction

The control of wind turbine generator systems (WTGS) has received a great deal of attention over the last years. It has been motivated both by the increasing interest in renewable energy due to environmental concerns and by the technical difficulty to deal with several wind regimes. We can find in the literature many approaches to tackle the control problem of a WTGS. Without attempting to be exhaustive, we can mention PID control (Jonkman et al., 2009), LQR control (Pintea et al., 2011) and LPV control (Bianchi et al., 2007; Bakka et al., 2014).

The purpose of the present paper is to design an optimal controller for an onshore three-bladed upwind variable-speed and variable-pitch WTGS subject to different wind regimes. We assume that the available control variables are the generator electromagnetic torque and the blade pitch angle. Furthermore, we assume that wind speeds are greater than the WTGS rated wind speed, and the controller objective is to keep the captured power within adequate boundaries in order to avoid excessive aerodynamical and mechanical loads.

In this paper the WTGS control problem is formally defined as a finite horizon quadratic optimal control problem of a discrete-time Markov jump linear system (MJLS) considering the case in which the Markov chain takes values in a general Borel space $\mathcal{S}$. It is assumed that the controller has access to an output variable as well as the jump parameter that represents the abrupt wind speed changes. The goal is to design a dynamic Markov jump controller such that the closed loop system minimizes the quadratic functional cost of the system over a finite horizon period of time. As shown in the principle of separation for discrete-time MJLS stated in (Costa and Figueiredo, 2016), the optimal controller is obtained from two $\mathcal{S}$-coupled difference Riccati equations, one associated to a filtering problem and the other one associated to a control problem in which the state variable is fully available. By $\mathcal{S}$-coupled it is meant that the difference Riccati equations are coupled via an integral over $\mathcal{S}$.

The paper is organized as follows. In Section 2 we introduce the basic notation and the control problem formulation. In Section 3 the optimal controller obtained in (Costa and Figueiredo, 2016) is presented. Section 4 is devoted to build a MJLS model and a control problem for the WTGS. In Section 5 we provide the results obtained from the simulation of the WTGS and some final comments.

2 Problem Formulation

The goal of this section is to present a general linear quadratic control problem of a MJLS with the Markov chain taking values in a Borel space. In Section 4 we will show how the control problem of a WTGS can be formulated as a linear quadratic control problem for a MJLS. First we introduce some notation.

Throughout the paper we fix a Borel space $\mathcal{S}$ and a σ-finite measure μ on $\mathcal{S}$. For $\mathbb{X}$ and $\mathbb{Y}$ Banach spaces we set $\mathbb{B}(\mathbb{X}, \mathbb{Y})$ for the Banach space of all bounded linear operators of $\mathbb{X}$ into $\mathbb{Y}$, with the uniform induced norm represented by $\|\cdot\|$. For simplicity we shall set $\mathbb{B}(\mathbb{X}) := \mathbb{B}(\mathbb{X}, \mathbb{X})$. As usual, $T \geq 0\, (T > 0)$ will mean that the operator $T \in \mathbb{B}(\mathbb{X})$ is positive semi-definite (positive definite, respectively). We say that $P = \{P(t); t \in \mathcal{S}\} \in \mathbb{H}_1^{n,m}$ if $P(\cdot) : \mathcal{S} \to \mathbb{B}(\mathbb{R}^n, \mathbb{R}^m)$ is measurable and $\|P\|_1 := \int_{\mathcal{S}} \|P(t)\|\mu(dt) < \infty$, and, similarly, that $P = \{P(t); t \in \mathcal{S}\} \in \mathbb{H}_{\sup}^{n,m}$ if $P(\cdot) : \mathcal{S} \to \mathbb{B}(\mathbb{R}^n, \mathbb{R}^m)$ is measurable and $\|P\|_{\sup} := \operatorname{ess\,sup}\{\|P(t)\|; t \in \mathcal{S}\} < \infty$. It is easy to see that $(\mathbb{H}_1^{n,m}, \|\cdot\|_1)$ and $(\mathbb{H}_{\sup}^{n,m}, \|\cdot\|_{\sup})$ are Banach spaces. For simplicity we will write $\mathbb{H}_1^n := \mathbb{H}_1^{n,n}$, $\mathbb{H}_{\sup}^n := \mathbb{H}_{\sup}^{n,n}$, and $\mathbb{H}_1^{n+} := \{P \in \mathbb{H}_1^n; P(t) \geq 0, \mu$ almost everywhere in $\mathcal{S}\}$, $\mathbb{H}_{\sup}^{n+} := \{P \in \mathbb{H}_{\sup}^n; P(t) \geq 0, \mu$ almost everywhere in $\mathcal{S}\}$. We say that $\widehat{P} \geq P$ if $\widehat{P}(t) \geq P(t)$ μ-almost everywhere in $\mathcal{S}$. The transpose of a matrix or vector will be denoted by $'$. The set of nonnegative integers will be denoted by $\mathbb{N}$ and the positive integers by $\mathbb{N}_1$. Fix a positive integer $T > 1$. The set of nonnegative integers less or equal to $T-1$ will be denoted by $\mathbb{T}$ and those greater than 1 and less or equal to $T-1$ by $\mathbb{T}_1$.

On a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ we consider a time-homogeneous Markov chain $\{\theta(k); k \in \mathbb{N}\}$ taking values in a Borel space $\mathcal{S}$ and with transition probability kernel $\mathcal{G}(\cdot|\cdot)$ having a density $g(\cdot|\cdot)$ with respect to a σ-finite measure μ on $\mathcal{S}$, so that for any $k \in \mathbb{N}$,

$$\mathcal{G}(\theta(k+1) \in C | \theta(k) = \ell) = \int_C g(t|\ell)\mu(dt). \tag{1}$$

Consider the following discrete-time MJLS

$$\begin{aligned} x(k+1) &= A(\theta(k))x(k) + B(\theta(k))u(k) \\ &\quad + G(\theta(k))w(k) \\ y(k) &= L(\theta(k))x(k) + H(\theta(k))w(k) \end{aligned} \tag{2}$$

with initial condition (x_0, θ_0) such that $E(x_0) := \zeta$, $E(x_0 x_0') := \mathbb{Q}_0$, and x_0, θ_0 are mutually independent. Notice that $E(\|x_0 - \zeta\|^2) = \operatorname{tr}(\mathbb{Q}_0 - \zeta\zeta') < \infty$. We denote by ν the initial probability measure on $\mathcal{S}$ for θ_0, so that for all $B \in \mathcal{B}(\mathcal{S})$,

$$\mathcal{P}(\theta_0 \in B) = \nu(B). \tag{3}$$

We assume that:

A0) The input sequence $\{w(k); k \in \mathbb{N}\}$ is a r-dimensional zero mean white noise sequence, (that is, $\{w(k); k \in \mathbb{N}\}$ is a sequence of independent identically distributed random vectors with $E(w(k)) = 0$ and $E(w(k)w(k)') = I$), and that $\{w(k); k \in \mathbb{N}\}$, the Markov chain $\{\theta(k); k \in \mathbb{N}\}$, and the initial state x_0 are mutually independent.

A1) $A \in \mathbb{H}_{\sup}^n$, $B \in \mathbb{H}_{\sup}^{m,n}$, $G \in \mathbb{H}_{\sup}^{r,n}$, $L \in \mathbb{H}_{\sup}^{n,p}$, $H \in \mathbb{H}_{\sup}^{r,p}$.

A2) $G(\ell)H(\ell)' = 0$ for every $\ell \in \mathcal{S}$.

A3) For some $c \geq 1$ we have that $g(\ell|t) \leq c$ for all ℓ, $t \in \mathcal{S}$.

The class of admissible controls with finite horizon T is defined next.

Definition 1 - Admissible Controls $\mathcal{U}$: *Choose $\widehat{x}(0) = \widehat{x}_0 \in \mathbb{R}^n$ deterministic and $\widehat{A}(k) \in \mathbb{H}_{\sup}^n$, $\widehat{B}(k) \in \mathbb{H}_{\sup}^{p,n}$ and $\widehat{C}(k) \in \mathbb{H}_{\sup}^{n,m}$ for each $k \in \mathbb{T}$. The class of admissible controllers $u = \{u(k); k \in \mathbb{T}\}$ associated to this choice will be defined for $u(k)$ obtained from the following Markov jump linear system:*

$$\begin{aligned} \widehat{x}(k+1) &= \widehat{A}(k, \theta(k))\widehat{x}(k) + \widehat{B}(k, \theta(k))y(k) \\ u(k) &= \widehat{C}(k, \theta(k))\widehat{x}(k) \end{aligned} \tag{4}$$

Thus the controller can choose the matrices $\widehat{A}(k) \in \mathbb{H}_{\sup}^n$, $\widehat{B}(k) \in \mathbb{H}_{\sup}^{p,n}$, $\widehat{C}(k) \in \mathbb{H}_{\sup}^{n,m}$ and the deterministic initial condition $\widehat{x}(0)$ in (4) in order to define the control $u(k)$. We have, from the hypothesis A1) and that $\widehat{A}(k) \in \mathbb{H}_{\sup}^n$, $\widehat{B}(k) \in \mathbb{H}_{\sup}^{p,n}$ and $\widehat{C}(k) \in \mathbb{H}_{\sup}^{n,m}$, that $E(\|x(k)\|^2) < \infty$ and $E(\|\widehat{x}(k)\|^2) < \infty$.

The following hypothesis is related to the quadratic optimal control problem to be defined in the sequel and imposes a strictly positive definite quadratic cost on the control variable $u(k)$:

A4) We assume that $C \in \mathbb{H}_{\sup}^{n,r}$, $D \in \mathbb{H}_{\sup}^{m,p}$, $D(\ell)'D(\ell) \geq \epsilon_D I$ for all $\ell \in \mathcal{S}$ and some $\epsilon_D > 0$, and $S \in \mathbb{H}_{\sup}^{n+}$.

The quadratic optimal control problem with partial information and finite horizon T we will consider is defined as follows:

Definition 2 - Control Problem with Partial Information: *Find $u \in \mathcal{U}$ that minimizes the quadratic cost $J(x_0, \theta_0, u)$ defined as:*

$$\begin{aligned} J(x_0, \theta_0, u) = \sum_{k=0}^{T-1} E(\|C(\theta(k))x(k)\|^2 \\ + \|D(\theta(k))u(k)\|^2) + E(x(T)'S(\theta(T))x(T)). \end{aligned} \tag{5}$$

The optimal cost is defined as

$$J(x_0, \theta_0) = \inf_{u \in \mathcal{U}} J(x_0, \theta_0, u). \tag{6}$$

Define the sequence $\{\pi(k); k \in \mathbb{N}\}$ as $\pi(0, \ell) := 1$,

$$\pi(k+1, \ell) := E(g(\ell|\theta(k))),\ k \in \mathbb{N}. \tag{7}$$

From (7) we have that for any $B \in \mathcal{B}(\mathcal{S})$ and $k \in \mathbb{N}$, $\mathcal{P}(\theta(k+1) \in B) = E(\mathcal{P}(\theta(k+1) \in B|\theta(k))) = E(\int_B g(\ell|\theta(k))\mu(d\ell)) = \int_B E(g(\ell|\theta(k)))\mu(d\ell) = \int_B \pi(k+1, \ell)\mu(d\ell)$. Thus, for $k \in \mathbb{N}$, $\pi(k+1)$ is the density function of $\theta(k+1)$ with respect to μ. From Assumption A3) and (7) we get that $\pi(k, \ell) \leq c$, so that $\pi(k) \in \mathbb{H}_{\sup}^{1+}$, $k \in \mathbb{T}$.

Our final assumption imposes a strictly positive condition on the covariance matrices of the observation noises, weighted in the Markov jump case by the density function of $\theta(k)$ with respect to μ for $k \in \mathbb{N}_1$.

A5) There exists $\epsilon_H > 0$ such that, for each $k \in \mathbb{T}$, $H(\ell)H(\ell)'\pi(k, \ell) \geq \epsilon_H I$ for all $\ell \in \mathcal{S}$.

We recall that hypothesis A0), A2), A4) and A5) are usually assumed in the LQG literature. Moreover, most of the transition density functions found in applications would satisfy the uniform boundedness condition required in A3).

3 Quadratic Optimal Control with Partial Information

In this section we present an optimal solution for the control problem stated in Definition 2. The optimal controller is derived from two $\mathcal{S}$-coupled Riccati difference equations, one of them associated to an optimum controller when the state variables are available, and the other one associated to an optimal filtering problem. This result is usually known as the "separation principle" and it will be of fundamental importance when dealing with the control problem for the WTGS to be presented in the next section.

3.1 Filtering and Control $\mathcal{S}$-coupled Riccati Difference Equations

We define next the filtering and control $\mathcal{S}$-coupled Riccati difference equations that will be useful when dealing with the control problem stated in Definition 2.

Set $Y(0) := (\mathbb{Q}_0 - \zeta\zeta') = E((x_0 - \zeta)(x_0 - \zeta)') \geq 0$, and for notational convenience we write $Y(0,\ell) = Y(0)$ for all $\ell \in \mathcal{S}$. Notice that $\|Y(0)\| \leq E(\|x_0 - \zeta\|^2) = \mathrm{tr}(\mathbb{Q}_0 - \zeta\zeta') < \infty$. The filtering $\mathcal{S}$-coupled Riccati difference equations are defined as follows:

Definition 3 - Filtering $\mathcal{S}$-coupled Riccati difference equations: *For $k \in \mathbb{T}$, $\ell \in \mathcal{S}$,*

$$\begin{aligned}Y(k+1,\ell) &:= \int_{\mathcal{S}} \Big[A(s)Y(k,s)A(s)' + G(s)G(s)' \\ &\cdot \pi(k,s) - A(s)Y(k,s)L(s)'R(k,s)L(s)Y(k,s) \\ &\cdot A(s)'\Big] g(\ell|s)\mu(ds), \qquad k \in \mathbb{T}_1, \end{aligned} \tag{8}$$

and

$$\begin{aligned}Y(1,\ell) &:= \int_{\mathcal{S}} \Big[A(s)Y(0)A(s)' + G(s)G(s)' \\ &- A(s)Y(0)L(s)'R(0,s)L(s)Y(0)A(s)'\Big] \\ &\cdot g(\ell|s)\nu(ds),\end{aligned} \tag{9}$$

where

$$R(k,\ell) := \Big(H(\ell)H(\ell)'\pi(k,\ell) + L(\ell)Y(k,\ell)L(\ell)'\Big)^{-1},$$

and

$$M(k,\ell) := -A(\ell)Y(k,\ell)L(\ell)'R(k,\ell). \tag{10}$$

For $k = T-1,\ldots,0$, define the control $\mathcal{S}$-coupled Riccati difference equations $X(k) \in \mathbb{H}^{n+}_{\sup}$ and feedback gains $F(k) \in \mathbb{H}^{n,m}_{\sup}$ as follows:

Definition 4 - Control $\mathcal{S}$-coupled Riccati difference equations:

$$\begin{aligned} X(T) = S, \quad X(k) = \mathcal{R}(X(k+1)),\\ F(k) = -\mathcal{K}(X(k+1)), \end{aligned} \tag{11}$$

where, for $Z \in \mathbb{H}^{n+}_{\sup}$ and $\ell \in \mathcal{S}$,

$$\begin{aligned}
\mathcal{E}(Z)(\ell) &:= \int_{\mathcal{S}} Z(t)g(t|\ell)\mu(dt),\\
\mathcal{Q}(Z)(\ell) &:= D(\ell)'D(\ell) + B(\ell)'\mathcal{E}(Z)(\ell)B(\ell),\\
\mathcal{K}(Z)(\ell) &:= \mathcal{Q}(Z)(\ell)^{-1}B(\ell)'\mathcal{E}(Z)(\ell)A(\ell),\\
\mathcal{R}(Z)(\ell) &:= C(\ell)'C(\ell) + A(\ell)'\mathcal{E}(Z)(\ell)A(\ell)\\
&\quad - A(\ell)'\mathcal{E}(Z)(\ell)B(\ell)\mathcal{Q}(Z)(\ell)^{-1}B(\ell)'\mathcal{E}(Z)(\ell)A(\ell).
\end{aligned}$$

3.2 The Separation Principle

In this subsection we state an optimal solution for the control problem with partial information presented in Definition 2.

Theorem 5 *An optimal solution for the problem posed in Definition 2 is obtained from the filtering $\mathcal{S}$-coupled Riccati difference equations presented in Definition 3 and the control $\mathcal{S}$-coupled Riccati difference equations stated in Definition 4. The gains $M(k,s)$ given by (10) and $F(k,s)$ given by (11) lead to the following optimal solution for $s \in \mathcal{S}$:*

$$\begin{aligned}
\widehat{A}^{op}(k,s) &= A(s) + M(k,s)L(s) + B(s)F(k,s),\\
\widehat{B}^{op}(k,s) &= -M(k,s),\\
\widehat{C}^{op}(k,s) &= F(k,s).
\end{aligned}$$

Proof: See (Costa and Figueiredo, 2016). □

4 Discrete-time MJLS Model and Control Problem for a WTGS

In Section 2 we have formally presented a model and a control problem for a discrete-time MJLS. Recalling these definitions in this section we build a MJLS model for the WTGS following similar steps as in (Lin et al., 2015, Section III). The model is obtained from several interconnected subsystems: a rotor, a pitch actuator, a mechanical, and an electrical subsystem. In this section we also present a control problem for the WTGS. The solution for this problem was stated in Theorem 5 and it will be used in the simulation described in the next section.

4.1 Rotor Subsystem

Based on Betz' theory, the relation between wind speed and mechanical power extracted from the wind is given by

$$P_r = \frac{\rho\pi R^2 V^3 C_P(\lambda,\beta)}{2}, \tag{12}$$

and the output mechanical torque of the wind turbine T_r can be calculated from the power P_r by

$$T_r = \frac{P_r}{\omega_r}, \tag{13}$$

where ρ is the mass density of air, R is the turbine rotor radius, V is the wind speed at hub height, C_P is the power coefficient, and ω_r is the hub angular velocity. Note that πR^2 is the swept area by the turbine blades. The tip speed ratio λ is defined as the ratio of the linear speed at the tip of blades to the speed of the wind and can be expressed as

$$\lambda = \frac{\omega_r R}{V}. \tag{14}$$

The power coefficient C_P can be expressed as a nonlinear function of both the blade tip speed ratio λ, which is determined by the blade design, and the blade pitch angle β. In this paper we will adopt an approximation as follows (Garcia-Sanz and Houpis, 2012, Chapter 12),

$$\begin{aligned} C_P(\lambda,\beta) &= c_1\left(\frac{c_2}{\lambda_i} - c_3\beta - c_4\right)\exp\left(-\frac{c_5}{\lambda_i}\right)\\ \lambda_i &= \left(\frac{1}{\lambda + c_6\beta} - \frac{c_7}{\beta^3+1}\right)^{-1}, \end{aligned} \tag{15}$$

where $c_1, c_2, \ldots, c_7$ are parameters that characterize a turbine. Recall that the main control objective when the wind speed is greater than the WTGS rated wind speed is to maintain the mechanical power around the rated power of the turbine.

4.2 Pitch Subsystem

The pitch angle actuator can be modeled as a first-order dynamic system given by

$$\dot{\beta} = \frac{1}{\tau_\beta}(\beta_{\text{ref}} - \beta), \tag{16}$$

where β_{ref} is the reference pitch angle and τ_β is the time constant of the actuator.

4.3 Mechanical and Electrical Subsystems

In the WTGS proposed in the present paper, we consider that the hub is fixed to the rotor shaft which drives the generator through a gearbox. We can model this transmission mechanism dynamics as a two mass model given by

$$J_r\,\dot{\omega}_r = T_r - n\,T_g, \tag{17}$$
$$J_g\,\dot{\omega}_g = T_g - T_e, \tag{18}$$
$$\omega_g = n\,\omega_r \tag{19}$$

where J_r is the inertia of the turbine rotor (blades, hub and rotor shaft), n is the gearbox ratio, T_g is the torque of the generator shaft, J_g is the inertia of the generator rotor (gearbox and generator shaft), and T_e is the electromagnetic torque. For simplicity, in (17)-(19) the torsional stiffness and damping associated with the wind turbine drive-train are ignored. Also for simplicity, it is assumed that the electric power produced by the WTGS is given by $P = \omega_g\,T_e$.

4.4 WTGS Nonlinear Model

Set the state variable $x_{n\ell} = [\beta \quad \omega_r]'$, the control input variable $u_{n\ell} = [\beta_{\text{ref}} \quad T_e]$, and the measured output variable $y_{n\ell} = [\omega_r]$. From (12)-(19) we get a nonlinear model for the WTGS given by

$$\dot{x}_{n\ell} = \begin{bmatrix} \frac{-x_{n\ell}(1)+u_{n\ell}(1)}{\tau_\beta} \\ \frac{T_r - n u_{n\ell}(2)}{J_r + n^2 J_g} \end{bmatrix} \tag{20}$$
$$y_{n\ell} = \left[x_{n\ell}(2)\right], \tag{21}$$

where T_r is given by (13). For a more complete model for the WTGS we refer to (Elkington et al., 2012; Bianchi et al., 2007).

4.5 WTGS Discrete-time MJLS Model and Control Problem

The wind speed signal V can be expressed in a given time instant as

$$V = \bar{V} + v, \tag{22}$$

where $\bar{V}$ is the mean wind speed, obtained as the average of the instantaneous speed over a time period, and v denotes the atmospheric turbulance (Bianchi et al., 2007). Notice that accurate wind speed measurements are rarely available in practice. It is possible to estimate the mean wind speed based on measurements of rotor speed, blade pitch angle and generator torque using a filter, e.g., an extended Kalman filter (Bakka et al., 2014).

From (12) and (13), the linearization of T_r around $\bar{V}$ results in the approximation

$$T_r \approx \bar{T}_r(\bar{V}) + \hat{T}_r(\bar{V})v, \tag{23}$$

where

$$\begin{aligned} \bar{T}_r(\bar{V}) &= T_r|_{V=\bar{V}} = \left.\frac{\rho\pi R^2 V^3 C_P(\lambda,\beta)}{2\omega_r}\right|_{V=\bar{V}} \\ \hat{T}_r(\bar{V}) &= \left.\frac{\partial T_r}{\partial V}\right|_{V=\bar{V}} = \frac{1}{2}\left[\frac{3\rho\pi R^2 V^2 C_P(\lambda,\beta)}{\omega_r}\right. \\ &\quad \left.- \rho\pi R^3 V \frac{\partial C_P(\lambda,\beta)}{\partial\lambda}\right]_{V=\bar{V}}. \end{aligned}$$

From (20) and (23) we get

$$\dot{x}_{n\ell} = \begin{bmatrix} \frac{-x_{n\ell}(1)}{\tau_\beta} \\ \frac{\bar{T}_r(\bar{V})}{J_r+n^2J_g} \end{bmatrix} + \begin{bmatrix} \frac{u_{n\ell}(1)}{\tau_\beta} \\ -\frac{n u_{n\ell}(2)}{J_r+n^2J_g} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\hat{T}_r(\bar{V})v}{J_r+n^2J_g} \end{bmatrix}. \tag{24}$$

The terms in (24) are associated with the state variable, control input variable and atmospheric turbulance respectively.

For a given mean wind speed $\bar{V}$ we can linearize (24) and (21) around the WTGS operation point $\bar{x} = [\bar{\beta} \quad \bar{\omega}_r]'$, such that $x = \bar{x} + \hat{x}$, and get

$$\begin{aligned} \dot{\hat{x}} &= \mathcal{A}(\bar{V})\,\hat{x} + \mathcal{B}\,u + \mathcal{G}(\bar{V})\,v \\ \hat{y} &= L\,\hat{x}, \end{aligned} \tag{25}$$

where $\mathcal{A}(\bar{V}) = \begin{bmatrix} -\frac{1}{\tau_\beta} & 0 \\ \frac{\frac{\partial \hat{T}_r(\bar{V})}{\partial \beta}\big|_{x=\bar{x}}}{J_r+n^2J_g} & \frac{\frac{\partial \hat{T}_r(\bar{V})}{\partial \omega_r}\big|_{x=\bar{x}}}{J_r+n^2J_g} \end{bmatrix}$, $\mathcal{B} = \begin{bmatrix} \frac{1}{\tau_\beta} & 0 \\ 0 & -\frac{n}{J_r+n^2J_g} \end{bmatrix}$, $\mathcal{G}(\bar{V}) = \begin{bmatrix} 0 \\ \frac{\hat{T}_r(\bar{V})}{J_r+n^2J_g} \end{bmatrix}$, $L = \begin{bmatrix} 0 & 1 \end{bmatrix}$. In the present paper, the operation point is defined such that ω_r is the rated hub angular velocity, and β is such that T_r and P_r assume their rated values.

Wind speed is subject to abrupt environmental changes. As a result of this variability, several linearized models are required to characterize the evolution of the WTGS. We will consider for simplicity just two wind conditions, named here as strong breeze and gale. This conditions represent respectively mean wind speeds that are slightly higher than rated speed, and pretty higher than rated speed. We will assume that the wind regime can be modeled by a time-homogeneous Markov chain $\{\theta(k); k \in \mathbb{N}\}$ with state space given by $\mathcal{S} = \{\{1\} \times [0,1]\} \cup \{\{2\} \times [0,1]\}$. For $\theta(k) = (i,t) \in \mathcal{S}$ the wind condition is represented by i, with $i = 1$ being strong breeze and $i = 2$ being gale, and the mean wind speed $\bar{V}$ depends on $\theta(k)$ as follows,

$$\bar{V}(\theta(k)) = V_{i1} + t(V_{i2} - V_{i1}), \tag{26}$$

where V_{11}, V_{12}, V_{21}, and V_{22} are constants. We consider that, for $\theta(k) = (i,t)$, we will have $\theta(k+1) = (j,U)$ with U being uniformly distributed in the interval $[0,1]$, and $j = 1$ with probability p_{i1}, and $j = 2$ with probability p_{i2} (of course $p_{i1} + p_{i2} = 1$, and $p_{ij} \geq 0$, $i,j = 1,2$).

Discretizing the model (25) using the zero-order holder method with sample time T_s, and considering output measurement noise, we get a MJLS model as in (2) for the WTGS that can be written as follows,

$$\begin{aligned} x(k+1) &= A(\theta(k))x(k) + B(\theta(k))u(k) \\ &\quad + G(\theta(k))w(k) \\ y(k) &= Lx(k) + Hw(k), \end{aligned} \tag{27}$$

where $A(\theta(k)) = e^{\mathcal{A}(\bar{V}(\theta(k)))T_s}$, $B(\theta(k)) = (\int_{\tau=0}^{T_s} e^{\mathcal{A}(\bar{V}(\theta(k)))\tau} d\tau)\mathcal{B}$, $G(\theta(k)) = [(\int_{\tau=0}^{T_s} e^{\mathcal{A}(\bar{V}(\theta(k)))\tau} d\tau)\mathcal{G}(\bar{V}(\theta(k))) \quad 0]$, and $H = [0 \quad h]$. We will assume the same hypothesis as in Section 2. It imposes that the sequence $\{w(k); k \in \mathbb{N}\}$ is a 2-dimensional zero mean white noise sequence.

Regarding the WTGS, we are interested in solving a control problem as the one stated in Definition 2, that is, we want to find $u \in \mathcal{U}$ that minimizes the quadratic cost $J(x_0, \theta_0, u)$ given by (5). The control objective is to keep the WTGS mechanical power held constant by controlling the reference blade pitch angle and the generator electromagnetic torque. The quadratic cost represents a compromise between two performance objectives: minimum tracking error and minimum control effort.

5 Simulation Results and Final Comments

In this section we provide the simulation results of the WTGS model obtained in Section 4.5.

The parameters considered in this simulation for the WTGS are based on (Jonkman et al., 2009; Zhang et al., 2014) and presented in Table 1. The WTGS rated wind speed is 11.4 m/s and we have adopted $V_{11} = 12$, $V_{12} = 14$, $V_{21} = 15$, and $V_{22} = 17$ in (26). The input noise sequence $\{v(k); k \in \mathbb{T}\}$ was assumed a gaussian white noise sequence. Parameters in the power coefficient approximation (15) where assigned to $c_1 = 0.39$, $c_2 = 116$, $c_3 = 0.4$, $c_4 = 5$, $c_5 = 16.5$, $c_6 = 0.089$, and $c_7 = 0.035$ (Garcia-Sanz and Houpis, 2012, Chapter 12). The sample time interval considered was $T_s = 6$ seconds, and the simulation lasted for 20 minutes, so that $T = 200$. Regarding the quadratic cost, for simplicity we have adopted, for all $\ell \in \mathcal{S}$, $C(\ell)^*C(\ell) = S(\ell) = I$ and $D(\ell)^*D(\ell) = I$ in (5).

Table 1: Parameters for the WTGS model

Parameter	Value	Unit
ω_r rated	1.2671	rad/s
T_r rated	3.946×10^6	N m
P_r rated	5.0×10^6	W
p_{11}	0.8	-
p_{22}	0.7	-
ρ	1.25	kg/m^3
R	61.5	m
τ_β	50×10^{-3}	s
J_r	3.68×10^7	kg m^2
J_g	5.30×10^2	kg m^2
n	97	-
T_s	6	s
h	0.1	-

We have solved equations (8)-(10) and (11) by considering a finite grid approximation for the space $\mathcal{S}$. Monte Carlo simulations of the closed loop system are presented in Figure 1. The figure contains 1000 possible trajectories for initial condition $x_0 = [0 \ \ 0]'$ and $\theta_0 = (1,0)$. The thick lines in the figure are the expected value trajectories. In Figure 1, we present data for the state variables related to the blade pitch angle β and to the hub angular velocity ω_r.

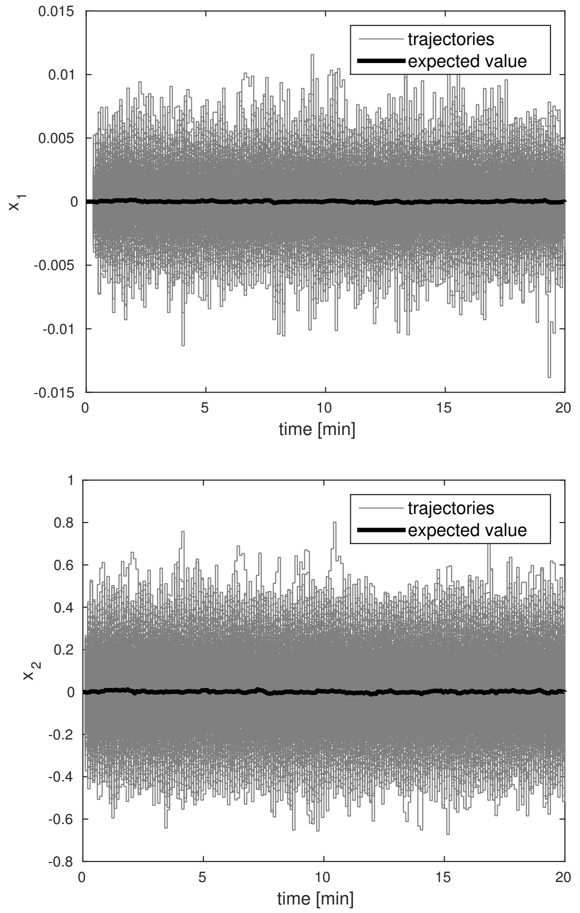

Figure 1: Monte Carlo simulations for the WTGS with partial observations.

The obtained results show that the approach proposed in this paper is able to keep the WTGS state variables within adequate boundaries. In this paper we have considered several modes of operation, each one associated to a different wind condition, but only one point of operation for the WTGS. The operation point was chosen to limit the captured power in order to avoid excessive aerodynamical and mechanical loads when wind speed is higher than the WTGS rated wind speed.

When the WTGS is subject to wind conditions such that wind speed is in the range between the WTGS cut-in and rated wind speed, the controller goal is the maximization of the power produced. In this case several operation points are necessary to model the WTGS. Thus, an interesting direction for future theoretical research is the study of optimal control problems for discrete-time MJLS subject to abrupt changes not only in its modes of operation but also in its operation points.

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