DISCRETE-TIME CONTROL DESIGN WITH COMPLEMENTARY STATE AND STATE-DERIVATIVE FEEDBACK

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Abstract— State-derivative feedback can be a useful strategy for the control of mechanical systems using accelerometers as sensors. Alternatively, when both state and state-derivative measurements are available, proportional and derivative state feedback can be employed. However, in practice, the full vectors of states and corresponding derivatives may not be directly available. Therefore, a control law based on partial state and state derivative measurements could be more convenient. In this context, the present paper proposes a method for the design of a discrete-time state and state derivative feedback controller, assuming that complementary components of the state and state derivative vectors are measured. The design is carried out in order to obtain nominal equivalence to a given discrete-time state feedback controller. A simulation case study involving an active mass damper system is presented for illustration.

Keywords— complementary state and state-derivative feedback, discrete-time control, active damping of structural vibrations

Resumo— A realimentação da derivada dos estados pode ser uma estratégia útil para o controle de sistemas mecânicos usando acelerômetros como sensores. Alternativamente, quando ambas as medidas dos estados e das derivadas dos estados estão disponíveis, pode-se empregar realimentação proporcional e derivativa dos estados. Porém, na prática, os vetores de estados completos e correspondentes derivadas podem não estar diretamente disponíveis. Desta forma, uma lei de controle baseada nas medidas parciais dos estados e derivadas dos estados poderia ser mais conveniente. Nesse contexto, o presente artigo propõe um método para o projeto de um controlador discreto com realimentação dos estados e da derivada dos estados, assumindo que componentes complementares dos vetores de estado e da derivada dos estados são medidos. O projeto é realizado visando obter equivalência nominal a um dado controlador discreto usando realimentação de estado. Um estudo de caso simulado envolvendo um sistema massa-amortecedor ativo é apresentado para ilustração.

Palavras-chave— realimentação complementar do estado e da derivada dos estados, controle a tempo discreto, amortecimento ativo de vibrações estruturais

1 Introduction

State derivative feedback (SDF) has been employed in various control problems, mainly involving the use of accelerometric sensors. Examples include (Duan et al., 2005; Abdelaziz, 2012; da Silva et al., 2013). In these cases, the state variables usually comprise displacements and velocities, which would need to be estimated by integrating the acceleration measurements. However, the accuracy of such estimates may be compromised by the presence of bias in the accelerometers, which would be an inconvenience for the implementation of conventional state feedback (SF) control laws.

SDF has been the subject of much research effort in the control literature. A pole placement method for linear systems using SDF was proposed in (Abdelaziz and Valášek, 2004). Eigenstructure assignment was presented in (Abdelaziz, 2012). The design of linear quadratic regulator (LQR) using SDF in reciprocal state space framework was described in (Duan et al., 2005). Stabilizability and stability robustness of SDF controllers were investigated in (Michiels et al., 2008). In (Basturk et al., 2015), an SDF adaptative controller was developed. Robust SDF design based on linear matrix inequalities for uncertain systems were proposed in (Assunção et al., 2007), (Faria et al., 2009),(da Silva et al., 2012).

The issue of proportional and derivative state feedback (PDSF) has also drawn much research attention, with studies including pole placement (Abdelaziz, 2015), normalization and stabilization for uncertain descriptor systems (Lin et al., 2005), nonlinear control with exact feedback linearization (Boukas and Habetler, 2004), decoupling of linear systems (Chu and Malabre, 2002) and H_∞ control for uncertain descriptor systems (Ren and Zhang, 2010), among others.

SDF formulations usually assume that all the derivatives of the states are available for feedback, whereas PDSF methods require both full-state and full-derivative state measurements. However, in practice, the full state vector and/or the full derivative state vector may not be directly available. Therefore, the combination of partial state and state-derivative measurements may be more convenient. Within this scope, Zhang et al. (2014) presented a method for partial eigenstructure assignment using acceleration and displacement feedback. An extension of this formulation was developed in (Zhang et al., 2016), using output signals consisting of linear combinations of accelerations and displacements.

In this context, the present paper proposes a method for the design of discrete-time control laws with complementary measurements of state and state derivative variables. For this purpose, it is assumed that the state components can be partitioned into two vectors x_1 and x_2, with x_1 and $\dot{x}_2$ available for feedback. The proposed method does not require measurements of the full vectors of states and/or corresponding derivatives, as in PSDF or SDF. On the other hand, it is more general than formulations solely based on displacements and accelerations, in that various combinations of partial displacement, velocity and acceleration measurements can be employed, if available. Moreover, the discrete-time nature of the proposed method is of value for direct implementation in digital computers. The design is carried out in order to obtain nominal equivalence to a given discrete-time SF controller, in a similar manner as developed in (Rossi et al., 2013) for discrete SDF controllers design. A simulation case study involving structural vibration damping is presented for illustration.

The remainder of this paper is organized as follows. Section 2 presents the proposed method for discrete-time complementary state and state-derivative feedback (CSSDF) design. Section 3 introduces the case study and Section 4 shows the results. Finally, concluding remarks are given in Section 5.

2 Proposed method

Consider a system described by a continuous-time model of the form

$$\dot{x}(t) = A_c x(t) + B_c u(t), \ \forall t \in \mathbb{R}^+ \tag{1}$$

with the state vector $x(t) \in \mathbb{R}^{n_x}$, the control input $u(t) \in \mathbb{R}^{n_u}$, and constant matrices $A_c \in \mathbb{R}^{n_x \times n_x}$, $B_c \in \mathbb{R}^{n_x \times n_u}$. Moreover, assume that a zero-order hold is employed to keep the control $u(t)$ constant between sampling times $t = kT$, $k \in \mathbb{Z}$, i.e.

$$u(t) = u(kT)^+, \ (kT)^+ \leq t \leq (k+1)T \tag{2}$$

where T is the sampling period. The superscript $+$ in (2) is employed to indicate that the control is updated immediately after the sensor measurements are acquired at each sampling time, as in (Rossi et al., 2013). Therefore, the state equation (1) at time $t = kT$ can be written as

$$\dot{x}(kT) = A_c x(kT) + B_c u((k-1)T)^+ \tag{3}$$

If the full state vector $x(kT)$ was available for feedback, a state feedback (SF) control law could be implemented as:

$$u(kT)^+ = -Lx(kT) \tag{4}$$

with a conveniently designed gain matrix $L \in \mathbb{R}^{n_u \times n_x}$.

The goal of this work consists of formulating a method for the design of discrete-time control laws with complementary state and state-derivative feedback (CSSDF) for system (1), such that the implemented control actions match those provided by a given SF control law of the form (4). For this purpose, consider that the state vector $x(kT)$ can be partitioned as

$$x(kT) = \begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix} \tag{5}$$

where $x_1(kT)$ and $\dot{x}_2(kT)$ are measured.

The following theorem presents the proposed formulation of the discrete-time CSSDF control law.

Theorem 1 . *Consider that a gain matrix $L \in \mathbb{R}^{n_u \times n_x}$ has been designed in order to obtain a suitable SF control law of the form (4) for system (3). Moreover, consider the state partition (5) and an associated partition of the A_c, B_c matrices as:*

$$A_c = \begin{bmatrix} A_{c,11} & A_{c,12} \\ A_{c,21} & A_{c,22} \end{bmatrix}, \ B_c = \begin{bmatrix} B_{c,1} \\ B_{c,2} \end{bmatrix} \tag{6}$$

Assume that matrix $A_{c,22}$ is non-singular. If the control law is implemented as

$$u(kT)^+ = -L\,M \begin{bmatrix} x_1(kT) \\ \dot{x}_2(kT) \\ u((k-1)T)^+ \end{bmatrix} \tag{7}$$

with

$$M = \begin{bmatrix} I & 0 & 0 \\ -A_{c,22}^{-1} A_{c,21} & A_{c,22}^{-1} & -A_{c,22}^{-1} B_{c,2} \end{bmatrix} \tag{8}$$

then the control values will match those provided by the state feedback law (4).

Proof: Under the assumption that the state vector $x(kT)$ and the matrices A_c and B_c of the state equation (3) are partitioned as (5) and (6), respectively, the state-derivative $\dot{x}_2$ can be written as:

$$\begin{aligned} \dot{x}_2(kT) =& A_{c,21} x_1(kT) + A_{c,22} x_2(kT) \\ &+ B_{c,2} u((k-1)T)^+ \end{aligned} \tag{9}$$

Assuming that $x_1(kT)$ and $\dot{x}_2(kT)$ are measured, the state $x_2(kT)$ can be obtained by reformulating equation (9) as

$$\begin{aligned} x_2(kT) =& A_{c,22}^{-1} [\dot{x}_2(kT) - A_{c,21} x_1(kT) \\ &- B_{c,2} u((k-1)T)^+] \end{aligned} \tag{10}$$

Using (9) in (7):

$$u(kT)^+ = -L\,M \begin{bmatrix} x_1(kT) \\ A_{c,21}x_1(kT) + A_{c,22}x_2(kT) + B_{c,2}u((k-1)T)^+ \\ u((k-1)T)^+ \end{bmatrix} \quad (11)$$

By substituting (8) in (11), it follows that

$$u(kT)^+ = -L \begin{bmatrix} x_1(kT) \\ w(kT) \end{bmatrix} \quad (12)$$

where

$$\begin{aligned} w(kT) &= -A_{c,22}^{-1}A_{c,21}x_1(kT) + A_{c,22}^{-1}A_{c,21}x_1(kT) \\ &+ x_2(kT) + A_{c,22}^{-1}B_{c,2}u((k-1)T)^+ \\ &- A_{c,22}^{-1}B_{c,2}u((k-1)T)^+ \\ &= x_2(kT) \end{aligned}$$

Therefore, it follows from (12) that

$$u(kT)^+ = -L \begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix} = -Lx(kT) \quad (13)$$

which corresponds to the control law (4). □

Remark 1 . *The proposed method requires the invertibility of the $A_{c,22}$ matrix, as stated in Theorem 1. In general, the SDF formulations proposed in the literature require the invertibility of the full A_c matrix, as can be seen for instance in (Abdelaziz and Valášek, 2004; Assunção et al., 2007; Duan et al., 2005; Moreira et al., 2010).*

3 Case study

Fig. 1 presents the active mass damper system under consideration. It consists of a building-like structure on top of which a cart (i.e. active mass) is driving along a linear track to dampen out the vibrations in the structure.

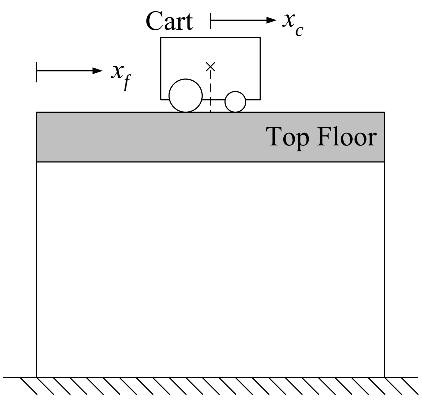

Figure 1: Active mass damper system.

The plant dynamics are described by the continuous-time state equation (1) with the state vector given by $x(t) = [x_c\,\dot{x}_c\,x_f\,\dot{x}_f]'$, where x_c and $\dot{x}_c$ denote the position and velocity of the cart with respect to the structure, and x_f and $\dot{x}_f$ denote the structure's top floor deflection (i.e. the horizontal displacement) and its velocity, respectively, and with the following A_c and B_c matrices (Quanser, Document Number 562):

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -18.7 & 279 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 5.98 & -336 & 0 \end{bmatrix},\; B_c = \begin{bmatrix} 0 \\ 3.00 \\ 0 \\ -0.96 \end{bmatrix} \quad (14)$$

Moreover, the control input is the motor voltage that drives the cart.

The challenge in designing a control system is that the top floor deflection x_f is not measured. Instead, the top floor is instrumented with an accelerometer to measure its acceleration $\ddot{x}_f$ relative to the ground. Additionaly, it is assumed that an encoder measures the cart position x_c and a tachometer measures the cart velocity $\dot{x}_c$.

In order to investigate the convenience of using a CSSDF controller when partial states and derivatives of other states are available for feedback, four cases were considered:

- Case 1: CSSDF controller, system under nominal conditions.
- Case 2: SF controller, system under nominal conditions.
- Case 3: CSSDF controller, constant bias of $+1m/s^2$ in the measured acceleration $\ddot{x}_f$.
- Case 4: SF controller, constant bias as in Case 3.

For the CSSDF controller, the partition (5) for the state vector $x(t) = [x_1'\; x_2']'$ was defined by setting $x_1 = [x_c\; \dot{x}_c]'$ and $x_2 = [x_f\; \dot{x}_f]'$, in order to implement the control law (7). In this case, $\ddot{x}_f$ is measured and $\dot{x}_f$ can be estimated by using a suitable integrating filter. On the other hand, the SF control law (4) is based on the states $x(t) = [x_c\,\dot{x}_c\,x_f\,\dot{x}_f]'$. In this case, not only $\dot{x}_f$ needs to be estimated, but also x_f. Therefore, two integrating filters are required.

In order to attenuate the effect of a sensor bias in the integration results, the integrating filters were designed in continuous-time as a series combination of an integrator and a high-pass filter, as follows:

$$F(s) = \frac{1}{s} \times \frac{\tau s}{\tau s + 1} = \frac{\tau}{\tau s + 1} \quad (15)$$

The parameter τ was chosen such that the filter $F(s)$ would not significantly deform the measurements of the structural vibrations. For this purpose, τ was chosen as $1/\tau << \omega_n$, where ω_n is the natural oscillation frequency of the system. The eigenvalues of matrix A_c in the continuous-time model are 0, -15.84 and $-1.42 \pm j17.00$. The oscillatory mode is associated to the complex conjugate eigenvalues, which present damping ratio $\zeta = 0.08$ and natural frequency $\omega_n = 17.1 rad/s$. Therefore, τ was chosen as $\tau = 0.3s$, which is approximately five times larger than $1/\omega_n$. It is worth noting that the system has small open-loop damping ($\zeta = 0.08$), which justifies the use of active vibration suppression.

For discrete-time implementation, the filter $F(s)$ was discretized by using Tustin's method (Franklin et al., 1997) with a sampling frequency of $100Hz$ ($T = 0.01s$), which is much larger than the natural frequency $\omega_n/(2\pi) = 2.7Hz$. As a result, the following z-domain transfer function was obtained:

$$F_d(z) = 0.4918 \times 10^{-2} \times \frac{z+1}{z-0.9672} \qquad (16)$$

A discrete-time linear quadratic regulator (DLQR) was designed as the state-feedback controller, with sampling period $T = 0.01s$. For this purpose, a discrete-time model of the form $x((k+1)T) = Ax(kT) + Bu(kT)^+$ was obtained by discretizing the state equation (1), with the matrices A_c and B_c given by (14), using the zero-order hold method, which resulted in:

$$A = \begin{bmatrix} 1.0000 & 0.0091 & 0.0131 & 0.0000 \\ 0 & 0.8298 & 2.5294 & 0.0131 \\ 0 & 0.0003 & 0.9835 & 0.0099 \\ 0 & 0.0543 & -3.2637 & 0.9835 \end{bmatrix} \qquad (17)$$

$$B = \begin{bmatrix} 0.0001 & 0.0273 & -0.0000 & -0.0087 \end{bmatrix}' \qquad (18)$$

The state and control weight matrices in the DLQR design were set to $Q = diag(10,10,10,10)$ and $R = 0.1$, respectively. Then, the following SF gain matrix L was obtained:

$$L = \begin{bmatrix} 9.0164 & 4.8663 & -62.9981 & -2.9756 \end{bmatrix} \qquad (19)$$

The CSSDF control law (7) was obtained by using the M matrix in (8) with the (A_c, B_c) matrices (14), partitioned as in (6).

In the simulations, the initial condition was set to $x(0) = [0\ 0\ 0.05\ 0]'$. Moreover, a small time delay of $T/100$ was introduced between sensor readings and control update to represent A/D conversion, computational processing and D/A conversion at each sampling time. All simulations were carried out by using the Matlab®/Simulink® software, with ode4 (4th order Runge-Kutta) solver and a fixed step size of $T/1000$.

4 Results and discussion

Fig. 4 presents the open-loop responses, under nominal operating conditions, of the cart position x_c (Fig. 4a), the top floor acceleration $\ddot{x}_f$ (Fig. 4b) and deflection x_f (Fig. 4c).

Fig. 3 show the time evolution of the cart position x_c, the structure's top floor acceleration $\ddot{x}_f$ and deflection x_f and the control variables using the CSSDF (Case 1) and the SF (Case 2) control laws in nominal operating conditions. As expected, by applying either CSSDF or SF control laws, the controlled system presents better performance (less oscillations and decreased settling time) for the top floor acceleration and deflection (Fig. 3b - 3c) compared to the open loop responses in Fig. 4. On the other hand, the SF control effort was larger compared to CSSDF (Fig. 3d), which resulted in a wider excursion of the cart (Fig. 3a).

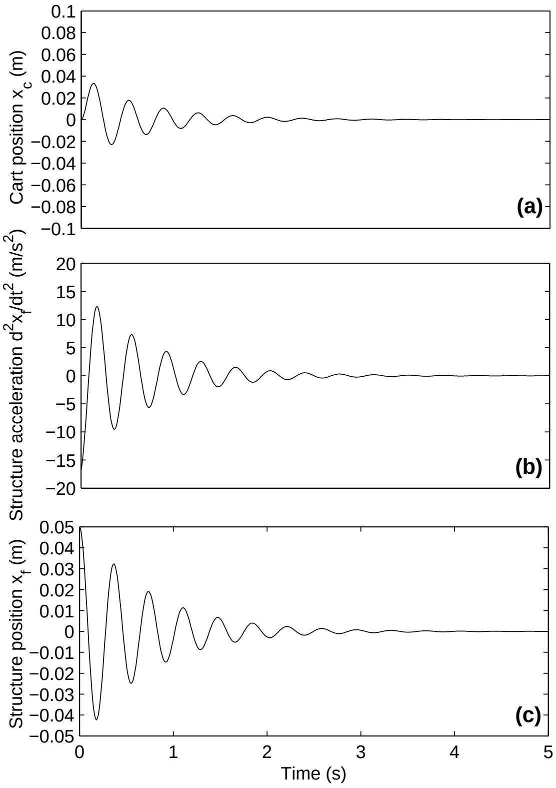

Figure 2: Open-loop responses. (a) Cart position. (b) Top floor acceleration. (c) Top floor deflection.

In the presence of a constant bias in the measurement of $\ddot{x}_f$, the simulation results by applying the CSSDF controller (Case 3) and the SF controller (Case 4) are presented in Fig. 4. For better clarity, the acceleration signals depicted in Fig. 4b correspond to the actual $\ddot{x}_f$ values, rather than the biased measurements employed for feedback. As can be seen, the top floor acceleration $\ddot{x}_f$ (Fig. 4b) and deflection x_f (Fig. 4c) responses are similar for both controllers, without significant differences with respect to the previous results in Fig. 3b and Fig. 3c). However, the SF control effort is now much larger compared to CSSDF, as shown in Fig. 4d. As a result, the cart moved farther away from its initial position, as shown in Fig. 4a. In practice, such an outcome would require the use of a much larger track for the cart, which may not be compatible with the physical dimensions of the structure. This would be a disadvantage of the SF control law compared to the proposed CSSDF implementation.

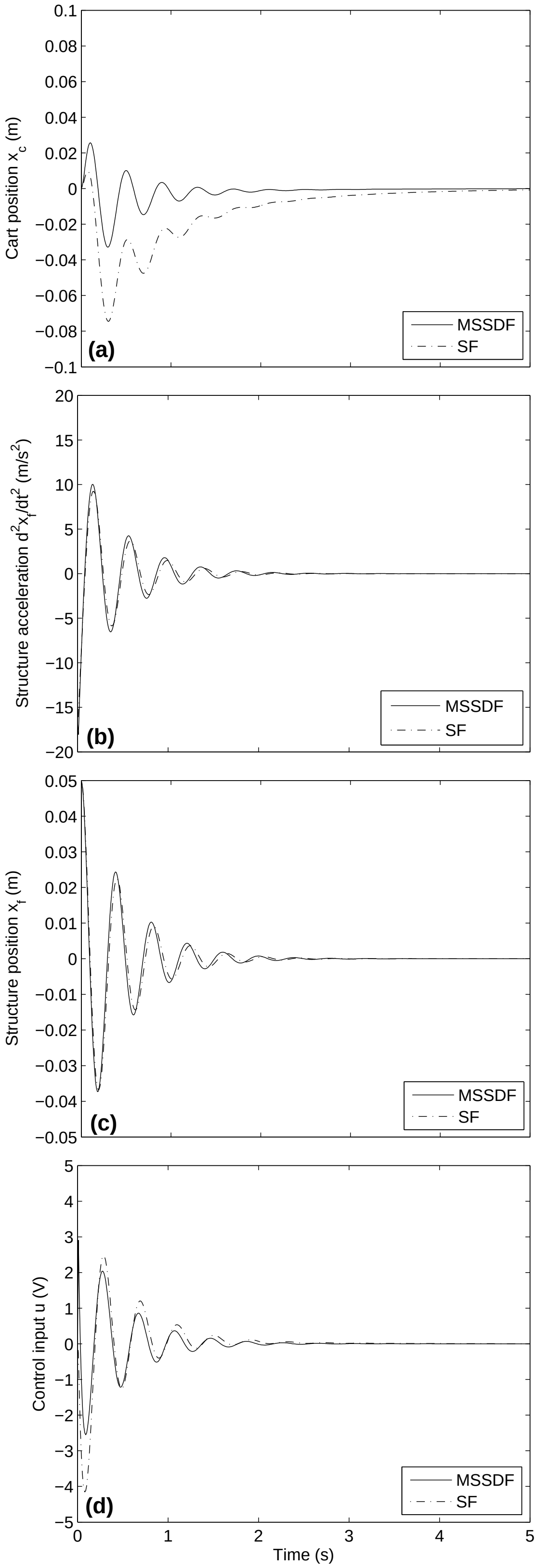

Figure 3: Closed-loop responses under nominal operating conditions. (a) Cart position (b) Top floor acceleration. (c) Top floor deflection. (d) Control input.

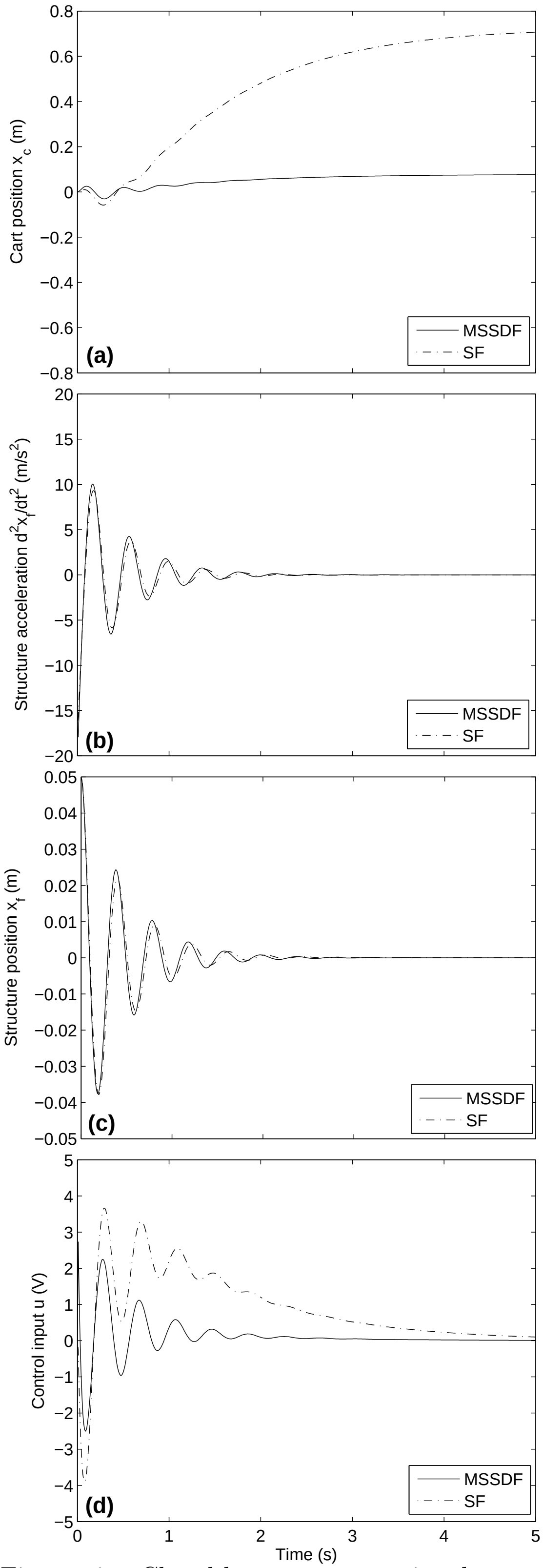

Figure 4: Closed-loop responses in the presence of measurement bias. (a) Cart position. (b) Top floor acceleration. (c) Top floor deflection. (d) Control input.

5 Conclusion

This paper proposed a method for the design of discrete-time control laws with complementary state and state derivative feedback (CSSDF), which is equivalent to a given discrete-time state feedback (SF) controller. A simulation case study involving the use of a moving cart as an active mass damper on top of a flexible structure was presented. In this case, the SF control law required the use of two integrating filters to estimate the velocity and displacement of the structure from the acceleration measurements. In contrast, the proposed CSSDF control law required only one integrating filter. As a result, the CSSDF control effort and the associated excursion of the moving cart were much smaller compared with SF, especially in the presence of bias in the acceleration measurements. Future studies could be concerned with the analysis of robustness of the CSSDF controller compared to the SF controller, in the presence of mismatch between the design model and the actual plant dynamics.

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