LONGITUDINAL AIRCRAFT RESPONSE ANALYSIS BASED ON SYSTEM IDENTIFICATION TECHNIQUES

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Abstract— The dynamic response of an aircraft in its longitudinal plane has two very distinctive modes: short-period and phugoid. The short-period has natural frequency and damping coefficient values which relates it to typical maneuvers in a normal operation. Thus, it is important its determination by flight tests to verify the aircraft flight quality. Generally, these flight tests are carried out through traditional methods. However, such procedures can be improved both for how the aircraft is excited as for the method used for analyzing the acquired data. This paper propose a new methodology to analyze the short-period mode through flight test techniques, making use of a 3-2-1-1 maneuver. For data reduction, system identification techniques are used, which are based on the data acquired during flight and on the aircraft equations of motion. The system identification method used here has several statistical properties of a "good estimator", thus, the estimated values converge in probability to the true values. Therefore, it is understood that the analysis of the aircraft response by the proposed method has a greater precision to represent the characteristics of the dynamic stability mode.

Keywords— Flight tests, system identification, short-period, aircraft response, output error method.

Resumo— A resposta dinâmica de uma aeronave em seu plano longitudinal possui dois modos distintos: curto-período e fugóide. O curto-período possui valores de frequência natural e coeficiente de amortecimento que o relacionam a manobras típicas em uma operação normal. Assim, é importante sua determinação através de ensaios em voo para verificar a qualidade de voo da aeronave. Geralmente esses ensaios são realizados através de métodos tradicionais. No entanto, tais procedimentos podem ser melhorados tanto na forma como a aeronave é excitada como no método utilizado para analisar os dados coletados. Esse artigo propõe uma nova metodologia para analisar o modo curto-período através de técnicas de ensaio em voo, utilizando a manobra 3-2-1-1. Para a redução dos dados, técnicas de identificação de sistemas são utilizadas, as quais são baseadas nos dados coletados durante o voo e nas equações de movimento da aeronave. O método de identificação de sistemas utilizado aqui possui diversas propriedades estatísticas de um "bom estimador", assim, os valores estimados convergem em probabilidade para os valores verdadeiros. Portanto, entende-se que a análise da resposta da aeronave pelo método proposto possui uma maior precisão para representar as características do modo de estabilidade dinâmica.

Palavras-chave— Ensaios em voo, identificação de sistemas, curto-período, resposta da aeronave, método do erro de saída.

1 Introduction

The response of an aircraft due to the pilot's commands or wind gusts are associated with flight safety factors and the crew's workload. In this way, the analysis of the dynamic behavior of an aircraft in flight is a very important task. Due to it, the natural frequency and the damping coefficient values must be known for each dynamic stability mode of an aircraft. In the beginning of a new aircraft design, these parameters can be calculated based on wind tunnel tests or on semi-empirical aerodynamic analysis. However, to verify the flight quality of an aircraft, it is necessary to make a dynamic stability analysis from data collected in flight tests (Kimberlin, 2003, Chapter 22).

General fixed-wing aircraft have five well-known dynamic stability modes, two longitudinal and three lateral-directional. The longitudinal responses are denoted long-period (or phugoid) and short-period, where the latter is characterized by higher values of natural frequency and damping coefficient when compared to the phugoid. Thus, the aeronautical regulations are more stringent about the short-period characteristics, since the phugoid has a slower response and is most easily controlled by the pilot. To the short-period analysis based on flight tests Kimberlin (2003, Chap. 23) and Ward (1996, Sec. 9.1) propose a simple methodology, which describes methods to excite the aircraft in this mode during flight test and to make the data reduction using graphical methods. This methodology has been widely applied and shows satisfactory results (Nicolosi, 2010), however, the graphical methods used in data reduction can show uncertainties, since they depends of the engineer's individual interpretation.

In this paper we propose a new methodology for short-period natural frequency and damping coefficient determination based on system identification method denominated Output Error Method (Jategaonkar, 2015; Klein, 2006), which is shown to be a better alternative for data analysis than the traditional graphical methods.

2 Methodology

Aircraft dynamic analysis during flight tests is done through data acquisition (linear speed, angular speed, Euler angles, etc.) while inputting specific maneuvers in the commands. Each maneuver aims to excite the aircraft in a specific dynamic mode, which can be interpreted from the reading of the acquired data.

This paper proposes a new methodology for aircraft short-period response analysis based on system identification techniques. Such techniques have statistical properties that produce results with higher reliability. The identification method used here is the Output Error Method (OEM), which use the flight data and the aircraft equations of motion to characterize the dynamic behavior of the vehicle. The proposed methodology is compared with the traditional approach by validation of the models against a different realization of the same maneuver.

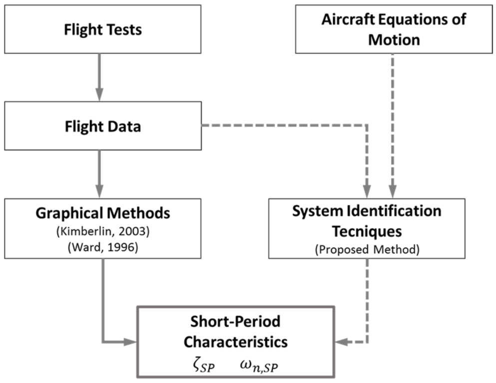

Figure 1. Comparison between methodologies for short-period analysis

Figure 1 summarizes flight data analysis using traditional graphical methods and using the proposed method based on system identification.

2.1 Aircraft Equations of Motion

The aircraft equation of motion describes the vehicle behavior in flight and is used for the parameter estimation process through the system identification methods. They are derived based on three reference frames, which are: Earth-based (or Navigation) frame, Body-based frame and Wind (or Stability) frame. The Navigation frame has its origin in the Earth surface and is adopted as an inertial reference. The Body-based axes has its origin at the aircraft gravity center (CG), as shown in Figure 2. The Wind axes, also with its origin at the CG, are related with the airspeed direction (Schmidt, 1998, Sec. 1.2.7; Pamadi, 1998, Sec. 4.2.4). The component of the linear velocity vector U, V and W and the components of the angular velocity vector P, Q and R are related to the Body-based axes X, Y and Z, respectively.

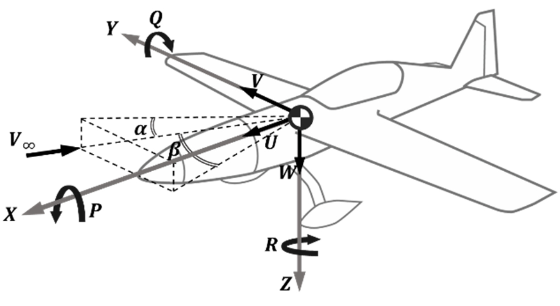

Figure 2. Linear velocity, angular velocity and aerodynamic angles

The aircraft equations of motion are obtained adopting the following assumptions: i) the aircraft is assumed as a continuous system and with constant mass; ii) the Earth curvature is neglected, that is, the Earth is assumed as a flat surface; iii) the aircraft is assumed to be a rigid body; iv) the aircraft mass distribution is assumed constant along the time, which implies that the moments and products of inertia can also be assumed as constant in time; v) the aircraft is symmetrical along the XZ plane, which allows decoupling of the motion in longitudinal and lateral-directional.

Modelling the aerodynamic, propulsive and gravitational forces and moments acting in the aircraft and assuming the condition of small perturbations in flight (Schmidt, 1998, Sec. 4.5 ; Pamadi, 1998, Sec. 4.3.4), we obtain the following linearized matrix formulation for the longitudinal motion,

$$\begin{Bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{Bmatrix} = [A] \begin{Bmatrix} u \\ \alpha \\ q \\ \theta \end{Bmatrix} + [B]\{\delta_E - \overline{\delta_E}\} \tag{1}$$

$$\{\dot{x}\} = [A]\{x\} + [B]\{\delta_E - \overline{\delta_E}\}$$

And

$$\{y\} = \{x\} + \begin{bmatrix} U_{eq} \\ \alpha_{eq} \\ Q_{bias} \\ \Theta_{eq} \end{bmatrix} \tag{2}$$

Where

$$\{x\} \triangleq [u \quad \alpha \quad q \quad \theta]^T$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = [B_{11} \quad B_{21} \quad B_{31} \quad 0]^T$$

In Eq.1, u, q and θ are the small perturbation in U, Q and Θ, respectively; and α is the small perturbation angle of attack. δ_E is the elevator deflection, which is responsible for the control in the longitudinal plane. $\overline{\delta_E}$, U_{eq}, α_{eq}, Q_{bias} and Θ_{eq} are the bias and equilibrium values. The elements of matrix A and B are composed by aerodynamic and propulsive dimensional coefficients (Pamadi, 1998, Sec. 6.2).

In stable aircrafts, the A matrix has two pairs of complex conjugate eigenvalues. Each pair represents one free response mode of the longitudinal dynamic stability. Generally, the long-period (phugoid) mode has low values of damping coefficient and natural frequency, on the other hand the short-period mode is characterized by relatively high values of damping coefficient (ζ_{SP}) and natural frequency ($\omega_{n,SP}$). Moreover, while the short-period is predominantly observed through α and q, the phugoid is observed through u

and θ (Schmidt, 1998, Sec. 6.2.1). One example of these roots are presented in the complex plane shown in Figure 3. The pilot can easily correct the phugoid, which has a slower response. However, the short-period, which has faster responses related to common maneuvers in flight, can turn the operation workload and unsafe during operation. Therefore, the short-period must be determined, e.g., using flight test techniques.

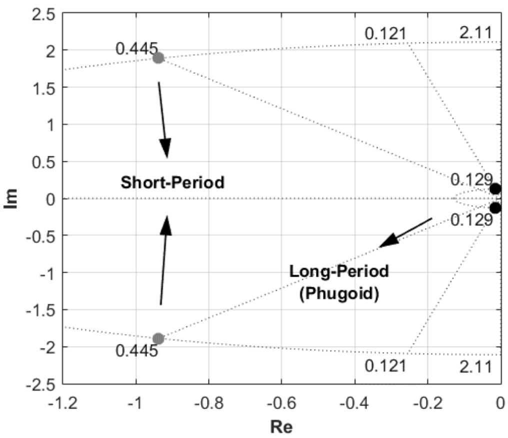

Figure 3. Locations of the phugoid and short-period roots

2.2 Flight Tests and Data Reduction

The short-period analysis based flight tests is used to verify the compatibility with the aircraft design data. A classic methodology used in some projects (Nicolosi, 2010) is proposed by Kimberlin (2003) and also by Ward (1996), where maneuvers to excite the short-period and graphical methods for data reduction are suggested. Kimberlin (2003) and Ward (1996) indicate the Doublet maneuver, since it has the advantage to excite the short-period, suppressing the phugoid mode. However, Jategaonkar (2015, Sec. 2.3.2) shows that the excitation 3-2-1-1 is preferable, since it is less sensitive to the Δt choice. Then, in this paper the 3-2-1-1 maneuver is chosen for short-period analysis.

The Maximum Slope Method (MS) is the main method for short-period data reduction suggested by Kimberlin (2003, Sec. 23.6). This method is based on the assumption that the natural response of the aircraft is dominated by a damped sinusoid (Eq. 3)

$$y(t) = Ke^{\zeta\omega_n t}\cos\left(\omega_n\sqrt{1-\zeta^2}t+\phi\right)+y_{eq} \quad (3)$$

Consequently, after the input returns to the equilibrium condition, the natural frequency $\omega_{n,SP}$ and damping coefficient ζ_{SP} can be obtained from a curve fit. The MS uses features of the graph, such as the maximum point, the minimum point and the maximum inclination of the tangent line (shown in Figure 4) to obtain such fit. The curve fit can also be done using optimization methods such as the Nonlinear Least Squares (NLS) method (Hamming, 1973), which is more easily automated and more robust to the presence of noise. Both methods were applied, with the parameters obtained with the MS fit being used as the initial guess for the NLS optimization.

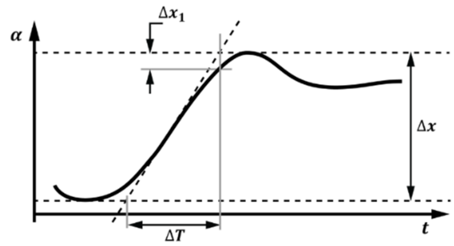

Figure 4. Measurements for the Maximum Slope Method

2.3 System Identification Methods

The short-period data reduction proposed in this paper is based on a system identification technique known as Output Error Method (Jategaonkar, 2015; Klein, 2006). This method is capable to estimate the parameters of the matrices A and B based on the data collected during the flight tests and on the aircraft equations of motion. Thus, the short-period characteristics are obtained through the evaluation of the A matrix eigenvalues.

In the Output Error Method (OEM) the system is assumed deterministic, that is, it is assumed that there is only noise (gaussian, white and zero mean) in the system output. The OEM is also called response-curve-fitting method, since it is based in the measurement of the distance between simulated responses of the system, for a given parameter group, and the output measured values. The vector of optimum parameters is that one which gives the highest probability to the measurements.

For the parameter estimation process, is used the system, which represents the longitudinal motion of the aircraft (Eq.1 and Eq. 2).

The cost function, based in Maximum Likelihood principle, is

$$J(\theta) = \frac{1}{2}n_y N + \frac{N}{2}\ln(\det(R)) + \frac{Nn_y}{2}\ln(2\pi) \quad (4)$$

Where n_y denotes the number of output equations, N the number of discrete time points at which the measurements are available and R the covariance matrix of the residuals, given by

$$R = \frac{1}{N}\sum_{k=1}^{N}[z(t_k)-y(t_k)][z(t_k)-y(t_k)]^T \quad (5)$$

In Eq. 5, $z(t_k)$ is the measured value at t_k ($k = 1,2,\ldots,N$), which is the time point at each measurement. The OEM estimation procedure is summarized in the following (Jategaonkar, 2015, Sec. 4.4):

1. An initial set of parameters (θ_0) is chosen. In this work, they come from the Equation Error

Method estimation (Jategaonkar, 2015, Chap.6).

2. System outputs y and residuals $(z - y)$ are computed. The measurement noise covariance matrix R is also estimated.
3. The cost function $J(\theta)$ is minimized with respect to the parameters θ by applying nonlinear optimization methods, such as Gauss-Newton.
4. Step 2 iterates until $J(\theta)$ reach its minimum value.

3 Results and Discussion

The methodology was applied to flight test data from the VFW-Fokker614/ATTAS (Advanced Technologies Testing Aircraft System) aircraft, from the DLR (German Aerospace Center). The data is available as supplementary material from Jategaonkar (2015), and was kindly ceded by the author. A data acquisition system was used to register the aircraft behavior over time during the execution of a 3-2-1-1 maneuver. The recorded data includes U, α, Q, Θ and δ_E. The same maneuver is performed twice at the same flight condition, creating two datasets, where one is used for the estimation and the other for the validation.

3.1 Graphical Analysis

In agreement with the Maximum Slope Method (Kimberlin, 2003; Ward, 1996), the short-period analysis must be done by the observation of the values of α in the aircraft response, since this mode can be predominantly represented by α behavior.

The measured values for the analysis (Figure 5) are described in Table 1.

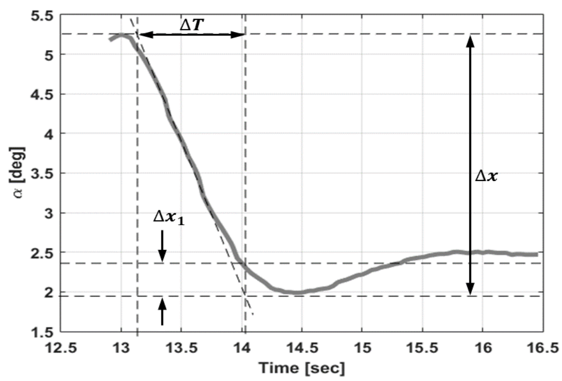

Figure 5. MS graphical features

Table 1. Measured MS graphical features.

Δx	Δx_1	ΔT
3.258	0.381	0.90

In the MS method the values of natural frequency and damping coefficient are determined using Figure 6, based on the data shown in Table 1.

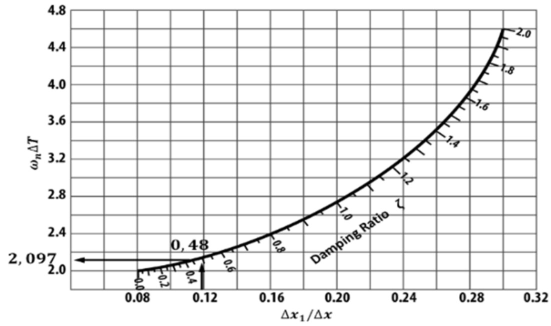

Figure 6. Damping coefficient and natural frequency determination. [Adapted from Ward (1996, Sec. 9.2)]

Then, we have that $\omega_n . \Delta T = 2.097$. The characteristics of the short-period based in the graphical approach are described in Table 2.

Table 2. Short-period characteristics (Maximum Slope Method).

ζ_{SP}	0.48
$\omega_{n,SP}$ [rad/s]	2.330

The MS method assumed that natural response of α is characterized by an exponentially damped curve, in the form of the Eq. 3, where $y(t)$ represents α in time. Figure 7 shows the raw data and the curve fitting obtained with MS method based on Eq. 3 and on the parameters in Table 2.

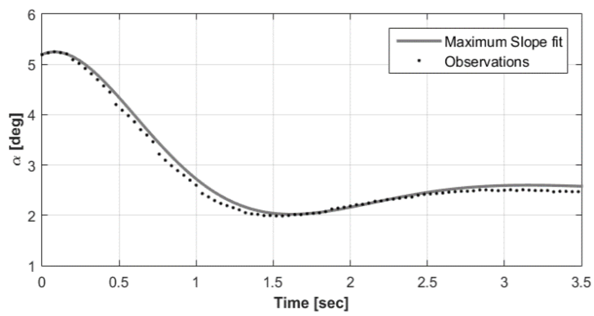

Figure 7. Maximum Slope Fit

The NLS fit to the same dataset is shown in Fig. 8 and Table 3.

Table 3. Short-period characteristics (NLS).

ζ_{SP}	0.5078
$\omega_{n,SP}$ [rad/s]	2.4762

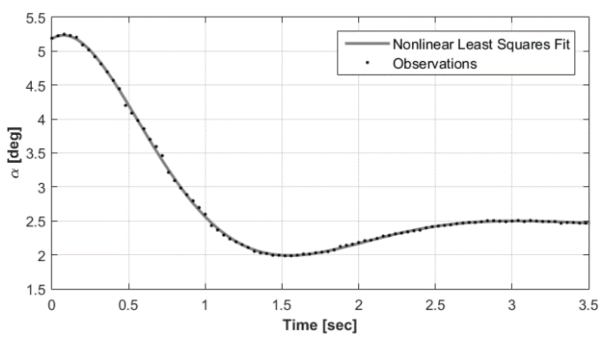

Figure 8. Nonlinear Least Squares Fitting

3.2 Parameter Estimation Analysis

In this paper, the proposed method for determining the characteristic of the short-period mode is based on the estimation of parameters A_{ij}, B_{ij}, $\overline{\delta_E}$, U_{eq}, α_{eq}, Q_{bias} and Θ_{eq} of the 4th order system (Eq. 1) from the data acquired in the tests. Figure 9 shows the result of the estimation for the first dataset.

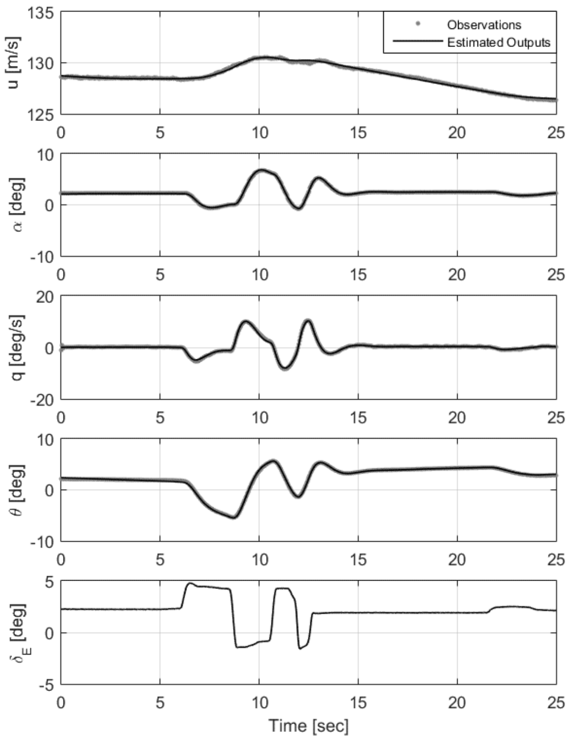

Figure 9. OEM estimation results

The A matrix estimated has two pair of complex conjugate roots. The pair representing the short-period is one that has the higher real absolute value. Thus, the roots representing the short-period are

$$s_{SP} = -1.2965 \pm 2.1895i$$

Therefore, the short-period natural frequency and damping coefficient are given in Table 3.

Table 4. Short-period characteristics (OEM).

ζ_{SP}	0.5095
$\omega_{n,SP}$ [rad/s]	2.5446

3.3 Validation

To validate and compare the different approaches, a second dataset of the same maneuver, under the same flight condition, was used. The root mean squared error of the angle of attack expected by the different methods was used to as the metric of comparison. For the MS and NLS methods, the amplitude K and phase angle ϕ of the damped sinusoid (Eq. 3) vary in different datasets. To be favorable to these methods, the values for these parameters which minimize the RMS error were obtained with the Gauss-Newton algorithm and used for in comparison. For the OEM method, the free simulation of the identified model was used, with the estimated parameter for matrices A and B and with a new estimation for the equilibrium parameters, since it is a new dataset.

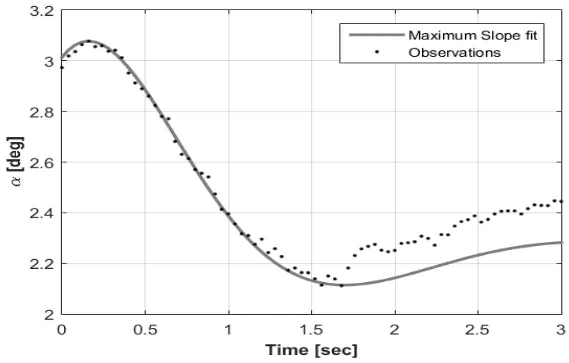

Figure 10. Maximum Slope Method Validation

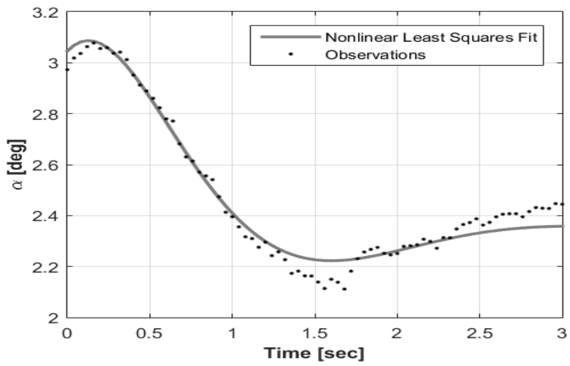

Figure 11. Nonlinear Least Squares Validation

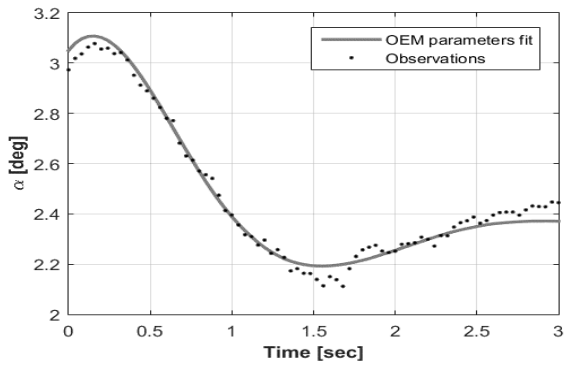

Figure 12. Output Error Method Validation

The Root Mean Squared Error for each method is shown in Table 5.

Table 5. Root Mean Squared Error for validation

Method	MS	NLS	OEM
RMS_{error}(°)	0.0911	0.0454	0.0364

We observe that the data analysis using graphical method (MS) can be improved by applying an exponential curve fitting procedure (NLS), instead of performing the analysis in the raw data. However, for both procedures, it is assumed that after the maneuver the pilot put the stick back to the equilibrium position, which is not always the regular situation. Besides, they assume a 2nd order model.

The use of the proposed method, where the short-period response characteristics are obtained by the OEM, we expect more reliable results, since this method can handle both process and measurement noise. Furthermore, it has several statistical properties of a “good estimator”, that is, the properties of maximum likelihood estimates implies that the estimated values converge in probability to the true values. Besides that, the property of asymptotic normality implies that the estimates obtained from different datasets corresponding to different experiments are clustered around the true value with a normal distribution (Jategaonkar, 2015). Therefore, we observe that the short-period characteristics from Table 4 must be closer to the real values, than those obtained by the other procedures.

4 Conclusion

General fixed-wing aircraft have two well-known modes of dynamic stability, among which is the “short-period”. The magnitude of its natural frequency and damping coefficient can be related to typical maneuvers during flight, which may render in an unsafe operation. Thus, it is important to analyze the characteristics of short-period mode through flight tests.

Some methods are proposed for both excitation of the aircraft in flight, and for the analysis of the acquired data. The Doublet maneuver is proposed for the mode excitation and the data reduction is based on the use of α values over time after application of the input. From these data it is possible to measure some parameters and determine the values of the natural frequency and damping coefficient for the short-period. The graphical analysis can be improved by using curve fitting, however, it is noted, even with such improvements, that these methods don't have the same probability qualities such as the system identifications methods.

In the proposed method, we have chosen to excite the aircraft using the maneuver 3-2-1-1, which due to its characteristics make it more indicated for testing. For data reduction, it is proposed the use of system identification techniques which are based on data obtained in the tests and on the aircraft equations of motion. Obtaining the short-period features using the OEM has several advantages when compared to the graphical method, which can be described as follows:

- The OEM is considered as a "good estimator", which essentially implies in unbiased estimates, asymptotic consistency, asymptotic normality and asymptotic efficiency (Jategaonkar, 2015, Sec. 4.3).
- The short-period analysis is not only based on α values, but on all variables for longitudinal equations of motion.
- A larger part of the data acquired in the maneuver is used.
- The system inputs (change in the aircraft controls) can be done more generally, and there is no need to return the stick to the initial equilibrium point after the application of the maneuver, which simplifies the execution of tests for the pilot.
- It can deal with higher fluctuations in the input.
- It can join different data sets to find an unique model
- The analysis of longitudinal motion through the system identification techniques allows the determination of all modes of dynamic stability (since the required maneuvers are applied), unlike the graphical method where only one mode is analyzed.
- The OEM can be used for analysis of high order / nonlinear models.

Therefore, it follows that the proposed techniques based on system identification results in a more accurate characterization of the short-period mode, generating more reliable results for the analysis of dynamic stability of the aircraft.

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